

# A Theory of Government Procrastination\*

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## Abstract

We present a theory to explain government procrastination as a consequence of its present-bias resulting from the political uncertainty in a multi-party political system. We show that under a multi-party political system the party in office tends to be present-biased. This may lead to procrastination of socially beneficial policies that carry upfront costs but yield long-term benefits. However, procrastination is often not indefinite even as we consider an infinite-horizon game. When the net social benefit is small, the policy can be procrastinated indefinitely, though there may co-exist some gradual implementation equilibria. When the net social benefit is intermediate in magnitude, there are a few types of procrastination equilibria, including gradual implementation. The theory predicts that a government with a more strongly predominant party tends to procrastinate less. When voters respond to a party's policy implementation by increasing its the future probability of being elected, the procrastination problem is mitigated but the pattern remains.

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# 1 Introduction

People often procrastinate when doing things that yield long-lasting benefits but carry an upfront cost, much to the detriment of their long-term interests. Quitting bad habits, such as smoking and drinking, is one such example. The literature on behavioral economics (e.g., Akerlof, 1991 and O'Donoghue and Rabin, 1999) explains this phenomenon by appealing to the existence of present-biased preferences. A present-biased individual's relative preference for payoff at an earlier date over that of a later date gets stronger as these dates approach. As a result, a task that appeared yesterday to be worth doing today becomes unworthy of doing when today arrives, possibly leading to repeated delay. A present-biased individual may procrastinate about completing a task forever, even though it is in her best long-term interest to complete the task immediately.

Similarly, it is often observed that politicians procrastinate about implementing socially beneficial policies that carry upfront costs but yield long-lasting benefits. For example, it is widely believed that the federal government and local governments of the U.S. underinvest in public infrastructure: many bridges seem to be in need of repair and many stretches of highway in need of renovation. The breaching of the dikes in New Orleans in 2005 caused by Hurricane Katrina might also be a case in point. The public was aware that the city was vulnerable to a severe storm such as Hurricane Katrina and investing in better defenses against storms was perceived to be a socially beneficial policy.<sup>1</sup> Yet the government did not act for many years. Politicians are also reluctant in many cases to raise income taxes even though it may benefit citizens in the long-run by helping to reduce the government deficit and hence lowering the long-term interest rate. Delay of trade liberalization, despite its long-term benefits to the country as a whole, can also be explained by the fact that the costs of resource reallocation (such as unemployment of workers) are incurred immediately while social benefits (of lower prices of imported goods for domestic consumers) are spread far into the future. Yet another prominent example of government procrastination is that of pension reform. As Feldstein (2005) states, “[m]any economists and policy analysts acknowledge the long-run advantages of shifting from a pay-as-you go [tax-financed] system to a mixed system

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<sup>1</sup>“Though everyone knew that the low-lying city was vulnerable to tempests and flooding, many who had lived there for years had experienced hurricane scares and smaller floods, and had come to view the risk of disaster with a sort of cheery aplomb.” (*The Economist*, September 2, 2005)

[that combines pay-as-you-go benefits with investment-based personal retirement accounts] but believe that the transition involves unacceptable costs.”<sup>2</sup> This might explain why many countries delay pension reform.

In this paper, we provide a theory to explain government procrastination about implementing a socially beneficial policy as a consequence of present-bias that results from political uncertainty inherent in a multiparty political system. We assume that in a two-party political economy model, both parties and citizens have the same time preferences, which are characterized by geometric discounting.<sup>3</sup> The effective discount rates, however, will be different in general between the two parties, and also from the citizens’ discount rate. Moreover, the parties’ effective discount rates between two consecutive periods are not constant over time; each party’s discount function exhibits hyperbolic discounting (see Laibson, 1997). Consequently, in a two-party political system the ruling party becomes present-biased and time-inconsistent. Present-bias arises because a party’s probability of getting elected in the future is less than one (but greater than zero) and because it puts more weight on the flow of net social benefit from the policy when it is in office than it would otherwise. As a result, the ruling party in a two-party system often procrastinates about implementing a socially beneficial policy that carries an upfront cost but yields long-term benefits.

Indefinite procrastination of socially beneficial policies can sometimes be explained by a model of a myopic government that cares more about current constituents and heavily discounts future unborn generations. In such a setting, the government has an incentive to procrastinate indefinitely about implementing a socially beneficial policy if and only if the government discounts future sufficiently more heavily than the citizens do. Since the government remains time-consistent, the policy is either immediately implemented in its entirety or procrastinated indefinitely, depending on the government’s discount rate. Thus, such models cannot explain why governments sometimes implement a policy only gradually. On the contrary, ours is not a model of myopia, but is a model of endogenous time-inconsistency of the political parties. A present-biased ruling party may not want to implement the policy now, but may wish that a future ruling party implement the policy; such time-inconsistency never occurs if governments are simply myopic. The outcomes of the model

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<sup>2</sup>The expressions in square brackets are added by the authors for clarity.

<sup>3</sup>By making this assumption, we rule out government procrastination resulting from differences in time preferences between the political parties and the citizenry or among the political parties themselves.

are also different from those of a myopic government, in that there exist equilibria in which, despite a certain degree of procrastination, a socially beneficial policy is implemented and carried out to completion in finite time.

We consider a government that is faced with the implementation of a policy that requires an upfront cost but yields long-term benefits to society. Some such policies are indivisible; they have to be completed in order to yield any benefits. Examples of indivisible policies include building a bridge and laying a railroad that connects two cities. We analyze the implementation of such an indivisible policy and find that various types of government procrastination can arise unless the policy cost is so small that both parties are willing to implement the policy whenever in office. We also consider the problem of policy implementation in the case of a divisible policy (or project); a policy is considered to be divisible if even partial implementation of it yields some benefits to society. A government can choose to partially open the country to international trade by eliminating or reducing some trade barriers, for example. In the case of balancing the budget, a government can choose to reduce the deficit somewhat but not entirely to a balanced budget. Our analysis shows that the possibility of partial implementation of the policy may enable the government to bypass the fate of indefinite procrastination of the policy even when the net social benefit is small (or equivalently the policy cost is large). Seen in this light, this paper identifies ruling parties' present-bias that is inherent in a two-party political system as a new source of gradualism in the literature on dynamic contribution to a public good.<sup>4</sup>

Our paper is related to the work of Alesina and Tabellini (1990). In their studies of government debt, they argue that the government saves too little, or accumulates too much debt, due to the political uncertainty caused by the two-party system. Amador (2003) and Aguiar and Amador (2011), using a similar framework, observes that the time-inconsistency with which a government with two symmetric parties is faced is equivalent to the problem faced by a single present-biased consumer whose present self plays a game with its future selves. The electorate is totally passive. In contrast, our paper emphasizes a dynamic game played by two asymmetric parties, with the electorate responding to the policy implementation by the parties. The current ruling party plays

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<sup>4</sup>Compte and Jehiel (2004), for example, obtain endogenous gradualism in a contribution game by assuming that raising a player's contribution in the negotiation phase increases the other player's outside option value.

the game with its own future selves as well as the future selves of the rival parties, taking into account the response of the electorate to their action. The following features distinguish our paper from those in the literature. First, to make the model as general as possible, we have introduced asymmetries in our model: (a) everything else being equal, the parties have asymmetric probabilities of being elected. We shall see that this gives rise to very different equilibrium outcomes than under symmetric countries, especially when one party is much more predominant than the other; (b) to capture incumbent advantage or disadvantage, we assume that the same party has different probabilities of being elected when it is an incumbent as opposed to being a non-incumbent.<sup>5</sup> Second, in the extension, we assume that there is give-and-take between the electorate and the political parties. In the extended model, we assume that a party's probability of being elected in the future increases when it implements the policy while in office. Despite introducing this assumption, the qualitative results remain the same as in the basic model but the parties both procrastinate less in the extended model. Third, we identify a cooperative equilibrium with gradual implementation resulting from the present-bias of the political parties. This form of procrastination is rarely discussed in the literature.

We set up a basic model of a two-party political system in Section 2. We show there that in an environment where two parties are elected into office according to a Markov process, the party in office will have hyperbolic discounting, and hence becomes present-biased, if both parties have a positive probability of being elected in each period and if each party can obtain utility flow from the policy only while in power. Section 3 shows that present-biased governments in some cases are tempted to procrastinate about implementing a socially beneficial policy that involves an upfront cost and yields long-term benefits. More specifically, unless the policy cost is small, the party in office wishes that a future government (which may be itself or its rival party) would implement the policy.

Section 4 derives the stationary subgame perfect equilibrium of the game played by the two

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<sup>5</sup>In our model, the election outcome is characterized by a Markov process, such that the current ruling party will be re-elected with an exogenous probability between 0 and 1. Moreover, that probability can be different for the two parties. Alesina and Tabellini (1990) and Amador (2003), however, assume that every party has an equal probability of being elected in every election. That is a special case of ours, when the probability of being re-elected equals 0.5 for both parties. Although Alesina and Tabellini (1990) mention in a footnote of their paper that the analysis can be extended to a similar framework to ours, they have not explored how the likelihood of being re-elected affects the government present-bias as much as we do in this paper.

parties.<sup>6</sup> We first consider the case in which the policy is indivisible. We show that if the policy cost is small enough, each party’s equilibrium strategy is to implement the policy whenever it is in office, and hence the policy is implemented immediately by the current ruling party. If the policy cost is large enough, on the other extreme, both parties procrastinate whenever they are in office, leading to indefinite procrastination. If the policy cost is in an intermediate range, however, there is a variety of equilibria that may arise. In all such cases, some delay in policy implementation is expected. In some equilibria, one party procrastinates whenever it is in office, while the other party always implements the policy when in power. There also exists a mixed-strategy equilibrium in which both parties randomize in policy decisions. This mixed-strategy equilibrium outcome is similar to the one that Alesina and Drazen (1991) obtain. They explain the delay in fiscal stabilization by a game of war of attrition between two heterogeneous socio-economic groups with conflicting distributional objectives. Stabilization is delayed because there is a stalemate in which the groups try to shift the burden of the policy change onto each other. Our game can also be viewed as a game of war of attrition between endogenously present-biased parties. Each ruling party has an incentive to procrastinate, hoping that the other party will implement in the future. In equilibrium, delay may arise either because one party always procrastinates or because both parties randomize.

In Section 4, we also examine the effect of a party’s predominance in election prospects on policy implementation. We find among other things that predominance of a party increases the hazard rate of policy implementation, particularly when the policy involves a large cost. This is because the predominant party will have more incentive to implement the policy, the greater the degree of its predominance over the other party. For example, China, which is under one-party rule, is well-known for its speedy construction of socially beneficial infrastructural projects such as the high-speed railway system: “[i]n 2008 China had only 649km of high-speed railway. It now has nearly 8,400km, four times as much as the next-largest network (Japan’s)” (*The Economist*, March 31, 2011). China also provided some other types of public goods at an amazing speed: “China’s

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<sup>6</sup>In our model, the stationary subgame perfect equilibrium is equivalent to the Markov perfect equilibrium, since we can define strategies such that Markov perfect equilibrium strategies do not vary with payoff-relevant states (such as the size of the remainder of the policy). More specifically, we define party  $i$ ’s strategy as a fraction  $a^i$  of the remainder of the policy, when we discuss the implementation of a divisible policy in Section 4.2.

rural health-insurance scheme, which in 2003 covered 3% of the eligible population, now covers 97.5%, according to official statistics” (*The Economist*, September 8, 2012).

We also show at the end of Section 4 that if the policy is divisible, there exists another possible form of implementation delay: if the policy is divisible, it may only be implemented gradually. We show that whenever the mixed-strategy equilibrium exists, there also exists a stationary subgame perfect equilibrium in which each party implements only a fraction of the remainder of the policy whenever it is in office. International trade has been liberalized only gradually for many countries, such as the United States, European countries, and Japan, where the cost of trade liberalization is considered to be large as they have large import-competing sectors, e.g., the agricultural sector. On the other hand, international trade has been substantially liberalized in small economies such as Singapore and Hong Kong, where the cost of trade liberalization is considered to be relatively small as they heavily specialize in export sectors and hence the import-competing sectors are small. This anecdotal evidence accords with the prediction of our analysis.

In Section 5, we turn our attention to identifying a self-enforcing, nonstationary, subgame perfect equilibrium with a trigger strategy. We find that even if the policy cost is too large for the two parties to implement the policy with stationary strategies, gradual implementation can be obtained in equilibrium if the policy is divisible and if the policy cost is not too large. In Section 6, we extend our basic model to the one in which the electorate rewards the party that implements the policy by increasing the likelihood of voting for the party. We find that political interaction of this type generally facilitates policy implementation, but the equilibrium configuration is qualitatively the same as in the basic model. Section 7 concludes the paper.

### **Other Related Literature**

There is a literature that theorizes that in a multiparty democracy, ruling parties cannot commit to implementing future policies. Krusell, Quadrini, and Ríos-Rull (1997) and Krusell and Ríos-Rull (1999) introduce political-economy considerations in a macroeconomic growth model to analyze endogenous public policy provision. In political-economy dynamic models of this kind, Hassler, Mora, Storesletten, and Zilibotti (2003) investigate a self-enforcing mechanism of the welfare state, while Klein, Krusell, and Ríos-Rull (2008) show that there is under-provision of public goods.

Bachmann and Bai (2010) show that contemporaneous correlation between output and government purchases is a declining function of the wealth bias in the political aggregation process. Battaglini and Coate (2008) extend the dynamic political economy theory to a model in which policy choices are made by a legislature, and show that equilibrium tax rates are too high, public good provision is too low, and debt levels are too high because redistributive considerations lead legislatures to be present-biased.

Besley and Coate (1998), Dixit, Grossman and Gul (2000), Hassler, Krusell, Storesletten and Zilibotti (2005), Acemoglu and Robinson (2008), Bai and Lagunoff (2008), and the recent literature on the endogenous voting franchise (see for example Acemoglu and Robinson (2006)) all look at how dynamic inconsistencies can arise from political uncertainty.

## 2 Basic Model: Preliminaries

There are two political parties,  $A$  and  $B$ , that seek control of the government. One of them is in office in period  $t \in \{0, 1, 2, \dots\}$ . Let each period be a term. Each party discounts future with a discount factor  $\delta \in (0, 1)$ , which is the same as the discount factor of a representative citizen.

The selection of the party in office in each election is characterized by a Markov process, such that the probability that a party is elected in the next period depends only on who is currently in office. Specifically, the probability that party  $A$  is re-elected in the next election if it is currently in office is  $\phi^A \in (0, 1)$ , while the probability that party  $B$  is re-elected if it is currently in office is  $\phi^B \in (0, 1)$ . Since there are only two parties, the probability that one party wins is equal to the probability that the other party loses. Therefore,  $\phi^A$  and  $\phi^B$  are the only parameters needed to fully describe the Markov process of the election outcome.

We assume in the basic model, analyzed in Sections 2 to 5, that the probability that a party is elected in the future is exogenous and therefore independent of whether the policy is implemented by the party or its rival. This assumption can be justified if a party's probability of being elected is determined by factors unrelated to the policy under discussion, and allows us to focus on the issue of policy implementation by a present-biased government. Moreover, it enables us to conduct a simple analysis concerning how the predominance of a party affects the policy implementation, when such



predominance is exogenously given.<sup>7</sup> In Section 6, we extend the basic model by assuming that a party's probabilities of being elected in the future are enhanced by implementing the policy today. This captures an important effect of a political interaction such that the electorate rewards a party that implements a socially beneficial policy by raising the likelihood of voting for the party.

The government is faced with the decision as to whether or not to implement a policy that involves an immediate cost of  $c$  but generates a constant benefit flow of 1 in every period after its implementation. We assume that the policy is socially beneficial: the discounted sum of the benefits outweighs the cost, i.e.,  $c < 1/(1 - \delta)$ , or equivalently  $(c - 1)/c < \delta$ . If the policy is divisible, the party in office can choose to implement only a fraction of the policy in its term so that a fraction  $a_t$  of the policy undertaken in period  $t$  poses an upfront cost  $a_t c$  to society while generating benefit flows of  $a_t$  in every period after its implementation.

The flow of utility enjoyed by citizens in period  $t$  is assumed to be equal to the flow of net social benefit resulting from the policy in that period, which is given by

$$u_t = \sum_{k=0}^t a_k - a_t c.$$

The first term on the right-hand side shows the flow of benefit that society enjoys in period  $t$  from the fraction of the policy that has been implemented, whereas the second term represents the flow of cost that society incurs from the part of the policy implemented in period  $t$ . Note that  $a_k$  takes any value in  $[0, 1]$  as long as  $\sum_{k=0}^t a_k \leq 1$  if the policy is divisible, while it takes 0 or 1 if the policy is indivisible. We shall assume that the policy is indivisible unless stated otherwise, although some important results are obtained in the case where the policy is divisible. The citizens' expected present discounted welfare in period  $t$  is given by

$$W_t = \sum_{k=0}^{\infty} \delta^k u_{t+k}.$$

We assume that the party in office in period  $t$  places a (normalized) weight of one on the flow of net social benefit in period  $t$  so that its utility in period  $t$  equals  $u_t$ , while the opposition party puts a weight of  $\alpha \in [0, 1)$  on the flow of net social benefit in the same period.<sup>8</sup> In other words, a party

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<sup>7</sup>It is often the case that party predominance is rather exogenous for historical reasons. Examples are the Liberal Democratic Party in postwar Japan, the People's Action Party in post-independence Singapore, and the Congress Party in post-independence India.

<sup>8</sup>The model can easily accommodate the case where the parties have different values of  $\alpha$ . We assume that they have the same value of  $\alpha$  only to simplify the exposition.

puts more weight on the flow of net social benefit when it is in power than it would otherwise. This differential weighting is motivated by the presumption that the ruling party derives some flow of private benefits spilled over from a flow of net social benefit during its term and that the flow of the private benefits is proportional to the flow of net social benefit. Provision of public infrastructures, such as dikes, bridges, highways, and the welfare system, helps the government to efficiently carry out various other future policies, such as industrial and tax policies. Thus, a party can reap additional benefits from the policy implementation when and only when it is in office in future periods. Another possible type of spillover is political capital that a ruling party may accumulate when the citizens' welfare is high. Outcomes of presidential elections in the United States and other countries often depend on economic performance at the time of election.<sup>9</sup>

Now, we are ready to show why each party will have present-bias. Let  $p_k^i$  denote the probability that party  $i$  in office in period  $t$  will also be in office in period  $t + k$ . Let  $u_{t+k}$  denote the flow of net social benefit in period  $t + k$  (as anticipated in period  $t$ ). Then, it follows from our discussion above that the welfare function for party  $i$  when it is in office in period  $t$  is given by

$$U_t^i = \sum_{k=0}^{\infty} \delta^k [p_k^i + (1 - p_k^i)\alpha] u_{t+k} \equiv \sum_{k=0}^{\infty} \beta_k^i u_{t+k}, \quad (1)$$

where  $\beta_k^i \equiv \delta^k [p_k^i + (1 - p_k^i)\alpha]$ . Although the utility flow accrued to ruling party  $i$  is the same as the flow of utility to the citizens in each period, the party's (effective) discount factor differs from that of the citizens in each period because of political uncertainty and spillover, as discussed above. We show that under a mild condition,  $\beta_{k+1}^i / \beta_k^i$  weakly increases with  $k$ , which directly implies by definition that the welfare function for the party in office exhibits generalized hyperbolic discounting and hence it has present-bias.

To this end, we first show that the probability that the party in office in period  $t$  will also be in office in period  $t + k$  weakly declines in  $k$ . Recall that  $\phi^i$  is the probability that ruling party  $i$  is re-elected in the next period. Define  $s \equiv \phi^A + \phi^B \in (0, 2)$ , which indicates the degree of incumbent advantage. The incumbent advantage is said to exist if the probability for a party to be elected in the next period is greater when it is currently in office than otherwise. Party  $i$  is

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<sup>9</sup>Aguiar and Amador (2011) also assume the differential weighting on social benefits of the same type as ours. See Aguiar and Amador (2011) for further justifications of this assumption.

elected in the next period with probability  $\phi^i$  if it is currently in office and  $1 - \phi^j$  if the rival party  $j$  is in office. Therefore, the incumbent advantage exists if and only if  $\phi^i > 1 - \phi^j$ , or equivalently  $s > 1$ . Henceforth, we assume that  $s \geq 1$ , i.e., there is no incumbent disadvantage (that arises when  $s < 1$ ).

Now, we derive below the probability that the current ruling party will also be in office  $k$  periods later and show that it converges to a steady state probability as  $k$  becomes large. For concreteness, let us suppose for now that party  $A$  is currently in office. First, note that the probability that party  $A$  will be in office  $k + 1$  periods later can be linked to the probability that party  $A$  is in office  $k$  periods later as follows:

$$\begin{aligned} p_{k+1}^A &= \phi^A p_k^A + (1 - \phi^B)(1 - p_k^A) \\ &= (1 - \phi^B) + (s - 1)p_k^A, \end{aligned}$$

with  $p_0^A = 1$ . We can solve this difference equation explicitly to obtain

$$p_k^A = \frac{(1 - \phi^B) + (1 - \phi^A)(s - 1)^k}{2 - s}, \quad (2)$$

for  $k \geq 1$ . Define  $p^A \equiv \lim_{k \rightarrow \infty} p_k^A$ . That is,  $p^A$  is the steady state probability that party  $A$  is in office. Then, it is clear that  $p^A = [\phi^A - (s - 1)] / (2 - s)$ . We can rewrite (2) as

$$p_k^A = p^A + (1 - p^A)(s - 1)^k \text{ for } k \geq 1. \quad (3)$$

Similarly, we have

$$p_k^B = 1 - p^A + p^A(s - 1)^k \text{ for } k \geq 1. \quad (4)$$

Note that  $p_0^A = p_0^B = 1$ .

As we see from (3) and (4) (together with  $1 \leq s < 2$ ) that  $p_k^A$  and  $p_k^B$  approach  $p^A$  and  $1 - p^A$ , respectively, as  $k$  becomes large. Without loss of generality, we assume that  $p^A \geq 1/2$ , or equivalently  $\phi^A \geq \phi^B$ . That is, we assume that party  $A$  is a (weakly) predominant party.

As we see from (3), the probability that party  $A$  is in office decreases over time from  $p_0^A = 1$  and converges to  $p^A$ . Incumbent party  $A$  has an advantage in the next election if  $s > 1$ . But this advantage diminishes, the further it is from the current period. The case where party  $B$  is currently

in office is similar; the probability that party  $B$  is in office decreases over time from  $p_0^B = 1$  to  $1 - p^A$ .

Recall from equation (1) that  $U_t^i = \sum_{k=0}^{\infty} \beta_k^i u_{t+k}$ , where  $\beta_k^i = \delta^k [p_k^i + (1 - p_k^i)\alpha]$ . Assuming that party  $A$  is in office in period  $t$ , the discount function for party  $A$  can therefore be written as  $\beta_0^A = 1$  and

$$\begin{aligned} \beta_k^A &= \delta^k [p_k^A + (1 - p_k^A)\alpha] \\ &= \delta^k \{\alpha + (1 - \alpha)[p^A + (1 - p^A)(s - 1)^k]\}, \end{aligned} \quad (5)$$

for  $k \geq 1$ . It is easy to see that if  $s > 1$ , equation (5) also applies to the case of  $k = 0$ , i.e., the equation yields  $\beta_0^A = 1$ . It directly follows from (5) that  $\beta_1^A/\beta_0^A = \delta[\alpha + (1 - \alpha)\{p^A + (1 - p^A)(s - 1)\}]$  and

$$\frac{\beta_{k+1}^A}{\beta_k^A} = \delta \left[ \frac{\alpha + (1 - \alpha)\{p^A + (1 - p^A)(s - 1)^{k+1}\}}{\alpha + (1 - \alpha)\{p^A + (1 - p^A)(s - 1)^k\}} \right], \quad (6)$$

for  $k \geq 1$ ; we have similar expression for  $\beta_{k+1}^B/\beta_k^B$ . It is readily seen that  $\beta_{k+1}^i/\beta_k^i$  weakly increases with  $k$  if  $0 \leq \alpha < 1$  and  $1 \leq s < 2$ . Indeed,  $\beta_{k+1}^i/\beta_k^i$  strictly increases with  $k$  if  $s > 1$ , while it increases from  $\beta_1^i \equiv \delta[\alpha + (1 - \alpha)p^A]$  for  $k = 0$  to  $\delta$  for  $k \geq 1$  if  $s = 1$ . Therefore, the welfare function for the ruling party  $i$  exhibits generalized hyperbolic discounting and hence the incumbent has present-bias if  $0 \leq \alpha < 1$  and  $1 \leq s < 2$ , which we assume throughout the analysis.

### 3 Temptation to Procrastinate

It has been shown in the literature that a present-biased individual exhibits time-inconsistent behavior, which includes inefficient procrastination of beneficial tasks that carry upfront costs but generate long-lasting future streams of benefits (see, for example, O'Donoghue and Rabin, 1999). In the current setting, the ruling party has present-bias, so it may have an incentive to procrastinate about implementing the policy that involves a large upfront cost.

We show here that there is a temptation for the current ruling party to procrastinate due to its present-bias. We define  $X_t^i$  as the present value of the utility flows obtained by ruling party  $i$  in period 0 if the policy is implemented in period  $t$  :

$$X_t^i \equiv \sum_{k=0}^{\infty} \beta_{t+k}^i - \beta_t^i c. \quad (7)$$

In particular, we have

$$X_0^i \equiv \sum_{k=0}^{\infty} \beta_k^i - c = 1 - c + \sum_{k=1}^{\infty} \beta_k^i. \quad (8)$$

We can use  $X_t^i$ , defined in (7), to write the expected welfare of the ruling party in period 0 (see (1) as

$$U_0^i = \sum_{t=0}^{\infty} a_t X_t^i,$$

where  $\sum_{t=0}^{\infty} a_t \leq 1$  and  $a_t \in \{0, 1\}$  if the policy is indivisible while  $a_t \in [0, 1]$  if it is divisible.

When does a ruling party have an incentive to procrastinate about implementing the socially beneficial policy? To answer this question, we compare the present values of utility flows obtained from policy implementation between two consecutive implementation periods  $t$  and  $t + 1$ . Party  $i$  that is in office in period 0 (weakly) prefers having the policy implemented in period  $t$  to having it implemented in period  $t + 1$  if and only if

$$\begin{aligned} X_t^i &\geq X_{t+1}^i \\ \Leftrightarrow \beta_t^i &\geq (\beta_t^i - \beta_{t+1}^i)c \\ \Leftrightarrow \frac{\beta_{t+1}^i}{\beta_t^i} &\geq \frac{c-1}{c}. \end{aligned} \quad (9)$$

The second inequality is easy to interpret: the ruling party in period 0 prefers having the policy implemented in period  $t$  to having it implemented in  $t + 1$  if and only if the loss from postponing implementation of the policy by one period, which equals  $\beta_t^i$ , is at least as high as the benefit from doing so, which equals  $(\beta_t^i - \beta_{t+1}^i)c$ . Recall that  $\beta_{t+1}^i/\beta_t^i$  weakly increases with  $t$  as (6) indicates. So if  $\beta_1^i/\beta_0^i \equiv \beta_1^i > (c-1)/c$ , inequality (9) holds for any  $t$ , and hence  $X_0^i$  is greater than  $X_t^i$  for any  $t \geq 1$ . In this case, the party in office in period 0 implements the entire policy. But if  $\beta_1^i < (c-1)/c$ , on the other hand, ruling party  $i$  in period 0 has an incentive to procrastinate since  $X_0^i < X_1^i$ . Figure 1 shows a possible sequence of  $X_t^i$  in such cases. The ruling party  $i$  in period 0 obtains the highest discounted utility when the policy is implemented in period 3 in this example.

Whether the party in office indeed procrastinates depends on the rival party's implementation strategy. The next section derives the stationary equilibrium of the policy implementation game.

## 4 Stationary Subgame Perfect Equilibrium

This section derives the stationary subgame perfect equilibrium of the infinite-horizon game played by the parties  $A$  and  $B$ . Party  $i$  can choose a policy only when it is in office. We define the action that the ruling party takes in a period by  $a$ , the fraction of the remainder of the policy the party implements. In section 4.2 and Section 5, we consider the case of a divisible policy in which  $a \in [0, 1]$ . In other sections, however, we analyze the case of an indivisible policy in which  $a \in \{0, 1\}$ . In this section, we derive the stationary subgame perfect equilibrium in which each party's equilibrium actions are constant over all histories.

### 4.1 Implementation of an Indivisible Policy

The analysis in the last section suggests (i) that if the policy cost  $c$  is small, the ruling party always implements the indivisible policy; (ii) that if  $c$  is in an intermediate range, the ruling party implements the policy if the rival party procrastinates, and vice versa; and (iii) that if  $c$  is large, the ruling party always procrastinates. We find that if  $c$  is in the intermediate range, there also exists a mixed-strategy equilibrium in which both parties randomize in the implementation decision (unless party  $A$  significantly predominates party  $B$  as discussed later). Since the analysis of the mixed-strategy equilibrium helps us identify the equilibrium in other cases, we first derive the mixed-strategy equilibrium that would arise if  $c$  is in the intermediate range.

#### Mixed-Strategy Equilibrium

A mixed-strategy equilibrium exists when each ruling party derives positive utility from implementing the policy immediately but would gain from procrastinating if it knows that the other party would implement the policy whenever it is in office. In such situations, there exists an equilibrium in which all ruling parties randomize their policy decisions, and each party is made indifferent between implementing the policy and procrastinating when in office.

We define  $\tilde{\beta}_k^i = \delta^k [1 - p_k^j + p_k^j \alpha]$  (where  $j \neq i$ ) as party  $i$ 's discount function  $k$  periods later given that party  $j$  is in office today. We also define

$$\tilde{X}_0^i = \alpha(1 - c) + \sum_{k=1}^{\infty} \tilde{\beta}_k^i \tag{10}$$

as the welfare of party  $i$  when party  $j$  implements the policy immediately given that party  $j$  is in office in period 0. Let  $\sigma^i$  denote the stationary probability that ruling party  $i$  implements the entire policy given that it has not been implemented. Let  $V^i$  ( $\tilde{V}^i$ ) denote the expected welfare of party  $i$  at the beginning of each period (before the election) when party  $i$  ( $j$ ) was in office in the last period, given that the policy has not been implemented.

Then, in the mixed-strategy equilibrium,  $V^A$  and  $\tilde{V}^A$  must simultaneously satisfy

$$V^A = \phi^A[\sigma^A X_0^A + (1 - \sigma^A)\delta V^A] + (1 - \phi^A)[\sigma^B \tilde{X}_0^A + (1 - \sigma^B)\delta \tilde{V}^A], \quad (11)$$

$$\tilde{V}^A = (1 - \phi^B)[\sigma^A X_0^A + (1 - \sigma^A)\delta V^A] + \phi^B[\sigma^B \tilde{X}_0^A + (1 - \sigma^B)\delta \tilde{V}^A]. \quad (12)$$

To understand equation (11), think of the existence of two lotteries. The first lottery determines who will be in office in this period (with probability  $\phi^A$  that  $A$  will be in office), and the second lottery determines whether the party in office in this period implements the policy, conditional on the identity of the party in office. If the ruling party implements the policy, then the game is over and party  $A$  enjoys flows of positive utility in all future periods (getting welfare of  $X_0^A$  or  $\tilde{X}_0^A$ ). Otherwise, the game proceeds to the next period (getting expected welfare of  $\delta V^A$  or  $\delta \tilde{V}^A$ ). The logic behind equation (12) is similar.

In the mixed-strategy equilibrium, ruling party  $A$  is indifferent between implementing and procrastinating, i.e.,  $X_0^A = \delta V^A$ . Substituting this into (12) and solving it for  $\tilde{V}^A$ , we obtain

$$\tilde{V}^A = \frac{(1 - \phi^B)X_0^A + \phi^B \sigma^B \tilde{X}_0^A}{1 - \delta \phi^B (1 - \sigma^B)}.$$

Then, we substitute this expression and  $X_0^A = \delta V^A$  into (11) to obtain

$$V^A = \frac{[\phi^A - \delta(s - 1)(1 - \sigma^B)]X_0^A + \sigma^B(1 - \phi^A)\tilde{X}_0^A}{1 - \delta \phi^B (1 - \sigma^B)}. \quad (13)$$

We apply  $X_0^A = \delta V^A$  one more time to equation (13) to get the implementation probability by ruling party  $B$  that renders party  $A$  indifferent between implementing the policy and procrastinating when in office:

$$\sigma^{B*} = \frac{[1 - \delta s + \delta^2(s - 1)]X_0^A}{\delta [(1 - \phi^A)\tilde{X}_0^A - \phi^B X_0^A] + \delta^2(s - 1)X_0^A}. \quad (14)$$

Similarly, we obtain the corresponding probability to be chosen by  $A$  to make  $B$  indifferent between

implementing the policy and procrastinating when in office:

$$\sigma^{A*} = \frac{[1 - \delta s + \delta^2(s - 1)]X_0^B}{\delta [(1 - \phi^B)\tilde{X}_0^B - \phi^A X_0^B] + \delta^2(s - 1)X_0^B}. \quad (15)$$

In the mixed-strategy equilibrium,  $\sigma^{B*}$  is chosen by ruling party  $B$  so that party  $A$  is indifferent between implementing the policy and procrastinating. Thus, in situations where ruling party  $A$ 's incentive to implement the policy increases due to some parametric changes (such as a fall in  $c$ ),  $\sigma^{B*}$  must increase to preserve this indifference. When  $\sigma^{B*}$  reaches 1, party  $A$  implements the policy when it is in office even if party  $B$  also does the same. On the other hand, if  $\sigma^{B*}$  equals 0, party  $A$  always procrastinates even if party  $B$  also never implements the policy.

### policy Costs and Equilibrium Outcome

Now, we are ready to derive the stationary subgame perfect equilibrium of the policy implementation game. As the above discussion suggests, the equilibrium feature depends on the values of the parameters of the model. Here, we derive the equilibrium for any possible value of the policy cost  $c$  that satisfies our assumption that  $c < 1/(1 - \delta)$ . Then, we examine how the likelihood of policy implementation changes with  $c$ .

To derive the stationary subgame perfect equilibrium, we first find how  $\sigma^{j*}$ , for  $j = A, B$ , shown in (14) and (15) changes with  $c$ . As we infer from the above argument,  $\sigma^{j*}$  takes a value in  $[0, 1]$  when  $c$  is in an intermediate range. We define  $\underline{c}^i$  as the policy cost  $c$  that satisfies

$$\frac{\tilde{X}_0^i}{X_0^i} = \frac{1 - \delta\phi^i}{\delta(1 - \phi^i)},$$

where  $X_0^i$  and  $\tilde{X}_0^i$  are given by (8) and (10), and  $\bar{c}^i \equiv \sum_{k=0}^{\infty} \beta_k^i$  as the cost  $c$  that satisfies  $X_0^i = 0$ . Then the following lemma shows that  $\sigma^{j*}$  takes a value in  $(0, 1)$  and is strictly decreasing in  $c$  on  $(\underline{c}^i, \bar{c}^i)$ , when  $j \neq i$ .

**Lemma 1** *For the threshold policy costs  $\underline{c}^i$  and  $\bar{c}^i$ , where  $\underline{c}^i < \bar{c}^i < 1/(1 - \delta)$ , we have  $\sigma^{j*} = 1$  if  $c = \underline{c}^i$  and  $\sigma^{j*} = 0$  if  $c = \bar{c}^i$ , where  $i \neq j$ . Moreover,  $\sigma^{j*}$  is strictly decreasing in  $c$  on  $(\underline{c}^i, \bar{c}^i)$ .*

Figure 2 visualizes Lemma 1, the proof of which is relegated to the Appendix.

Lemma 1 is useful in characterizing party  $i$ 's stationary best-response strategy. The lemma shows that  $\sigma^{j*} = 1$  if the policy cost is so small that  $c \leq \underline{c}^i$ , where  $j \neq i$ . That is, party  $i$  always



implements the policy when it is in office even though the other party  $j$  also implements the policy whenever it is in office. If  $c \geq \bar{c}^i$ , on the other extreme,  $\sigma^{j*} = 0$  so that party  $i$  never implements the policy regardless of the rival party's implementation strategy. If  $c$  is in the intermediate range such that  $\underline{c}^i < c < \bar{c}^i$ , the lemma shows that  $0 < \sigma^{j*} < 1$ , so party  $i$  implements the policy if the rival party  $j$  procrastinates, and vice versa. Moreover, party  $i$  is indifferent between implementing the policy and procrastinating if the rival party  $j$  implements the policy with probability  $\sigma^{j*}$ . We record this finding as a lemma.

**Lemma 2** *Party  $i$  always implements the policy when it is in office if  $c \leq \underline{c}^i$ , while it always procrastinates if  $c \geq \bar{c}^i$ . If  $\underline{c}^i < c < \bar{c}^i$ , party  $i$  (i) always implements the policy if the rival party  $j$  always procrastinates; (ii) always procrastinates if party  $j$  always implements the policy; and (iii) is indifferent between implementing the policy and procrastinating if party  $j$  implements the policy with probability  $\sigma^{j*}$  whenever it is in office.*

In order to fully characterize the stationary equilibrium of this basic model with the indivisible policy, we then show that the predominance of party  $A$ , as represented by  $\phi^A > \phi^B$ , induces the  $\sigma^{B*}$  schedule to be located to the right of the  $\sigma^{A*}$  schedule as depicted in Figure 2. The following lemma presents this feature in a different way.

**Lemma 3** *When  $\phi^A > \phi^B$ , we have  $\sigma^{A*} < \sigma^{B*}$  whenever either one of them takes a value in  $(0, 1)$ .*

The intuition for this lemma is simple, though a rigorous proof involves some computation and is therefore relegated to the Appendix. Since the predominant party (party  $A$ ) discounts future effectively less than the predominated party (party  $B$ ), party  $A$  derives more utility from the future flow of social benefits from the policy than does party  $B$ . Therefore, the probability that party  $B$  would implement the policy in order to make party  $A$  indifferent between implementing the policy and procrastinating is higher than the probability that party  $A$  would implement the policy in order to make party  $B$  indifferent between implementing and procrastinating. That is why we have  $\sigma^{A*} < \sigma^{B*}$  whenever they are different.

Having derived each party's best response to the other party's policy choice, we can now characterize the stationary subgame perfect equilibrium. Suppose first that the predominance of party

$A$  is not significant, so that  $\underline{c}^A < \bar{c}^B$ , as depicted in Figure 2. As we have seen, each party's best-response strategy depends on the policy cost  $c$ . The upper panel of Figure 3 depicts each party's best-response strategies that reflect Lemma 2 and Lemma 3 for all possible values of  $c$ .

Then it follows immediately that there are five types of equilibrium, as shown in the lower panel of Figure 3; which type of equilibrium arises depends on  $c$ .

- (i) If  $c \leq \underline{c}^B$ , there is a unique stationary subgame perfect equilibrium in which both parties implement the policy whenever they are in office. The policy is immediately implemented in this case.
- (ii) If  $\underline{c}^B < c \leq \underline{c}^A$ , there is a unique stationary subgame perfect equilibrium in which party  $A$  implements the policy while party  $B$  procrastinates. The policy is implemented when party  $A$  is elected for the first time.
- (iii) If  $\underline{c}^A < c < \bar{c}^B$ , there are three stationary subgame perfect equilibria:
  - (a) party  $A$  implements the policy while party  $B$  procrastinates,
  - (b) party  $A$  procrastinates while party  $B$  implements the policy,
  - (c) party  $A$  and party  $B$  implement the policy with probabilities  $\sigma^{A*}$  and  $\sigma^{B*}$ , respectively.

In equilibrium (c), the policy is implemented in finite time with probability 1.

- (iv) If  $\bar{c}^B \leq c \leq \bar{c}^A$ , there is a unique stationary subgame perfect equilibrium in which party  $A$  implements the policy while party  $B$  procrastinates. The policy is implemented when party  $A$  is elected for the first time.
- (v) If  $c > \bar{c}^A$ , there is a unique stationary subgame perfect equilibrium in which both parties procrastinate. The policy will not be implemented in this case.

It is worth mentioning that although the subgame perfect equilibria are the same in type (ii) and type (iv), the reasons why party  $A$  implements the policy while party  $B$  procrastinates are different between the two cases. In type (ii) equilibrium, party  $A$  implements the policy because it is in its best interest to do so regardless of party  $B$ 's implementation strategy. In type (iv)

equilibrium, on the other hand, party  $A$  implements the policy because the policy would never be implemented otherwise.

As the predominance of party  $A$  increases, party  $A$ 's incentive to implement the policy increases (and hence  $\underline{c}^A$  rises) while party  $B$ 's incentive to do so decreases (and hence  $\bar{c}^B$  falls). Therefore, if the predominance of party  $A$  is strong enough, equilibrium type (iii) disappears.

We summarize these findings in the following proposition.

**Proposition 1** *If the policy cost of the policy is small, the policy is immediately implemented despite the fact that both parties are present-biased. If the cost is high, neither party implements the policy. If the cost is in the intermediate range, some delay in implementation is expected. The delay may arise because one of the two parties always procrastinates when in office or because both parties adopt a mixed strategy as to whether or not they implement the policy when they are in office.*

Roughly speaking, the hazard rate of policy implementation decreases with the policy cost  $c$ : the hazard rate is 1 if  $c$  is small, positive but smaller than 1 if  $c$  is in the intermediate range, and 0 if  $c$  is large. But if we look closely at the equilibrium associated with the intermediate range of  $c$ , we recognize that the hazard rate is not monotonically decreasing in  $c$  if, for example, the equilibrium in which party  $A$  procrastinates while party  $B$  implements the policy is selected when  $c \in (\underline{c}^A, \bar{c}^B)$ . When  $c$  is in the intermediate range, each party procrastinates if the other party implements the policy. The hazard rate of policy implementation depends on which party is to procrastinate.

### Effects of party predominance

Let us examine how the existence of a party's predominance affects the policy implementation. Here, we assume that there is no incumbent advantage in elections, i.e.,  $s = 1$ , so that we do not have to distinguish whether party  $A$  or party  $B$  was in office in the last period when we discuss the effect of party  $A$ 's predominance on the hazard rate of policy implementation. Under this simplifying assumption, we only need to examine the effect of an increase in  $p^A$  on the hazard rate. We also assume that the opposition party does not derive any utility from the current flow of social benefit from the policy, i.e.,  $\alpha = 0$ , to simplify the exposition. The basic message from the

following analysis remains valid even without these additional assumptions, which are made only for the current discussion of the effect of a party's predominance on the hazard rate.

When  $s = 1$  and  $\alpha = 0$ , we have  $p_k^A = p^A$  and  $p_k^B = 1 - p^A$  for any  $k \geq 1$ . Therefore, we have

$$X_0^A = 1 - c + \frac{\delta p^A}{1 - \delta},$$

$$\tilde{X}_0^A = \frac{\delta p^A}{1 - \delta}.$$

We substitute these equations,  $s = 1$ , and  $\alpha = 0$  into (14) to obtain

$$\sigma^{B*} = \frac{\delta p^A - (1 - \delta)(c - 1)}{\delta(1 - p^A)(c - 1)}.$$

Party  $A$ 's counterpart can be similarly derived as

$$\sigma^{A*} = \frac{\delta(1 - p^A) - (1 - \delta)(c - 1)}{\delta p^A(c - 1)}.$$

As the predominance of party  $A$  increases, i.e.,  $p^A$  increases,  $\sigma^{B*}$  increases while  $\sigma^{A*}$  decreases. As we can infer from Figure 2, it entails increases of the cost thresholds  $\underline{c}^A$  and  $\bar{c}^A$  for party  $A$  and decreases of the thresholds  $\underline{c}^B$  and  $\bar{c}^B$  for party  $B$ . Those changes are indicated by the arrows in the lower panel of Figure 3. We find that regions (i), (iii), and (v) shrink while regions (ii) and (iv) expand. Indeed it is readily shown that

$$\underline{c}^A = \frac{1}{1 - \delta p^A}, \quad \bar{c}^A = \frac{1 - \delta(1 - p^A)}{1 - \delta},$$

$$\underline{c}^B = \frac{1}{1 - \delta(1 - p^A)}, \quad \bar{c}^B = \frac{1 - \delta p^A}{1 - \delta},$$

and that if the parties become so asymmetric that  $p^A > \bar{p}$ , where  $\bar{p} = (1 - \sqrt{1 - \delta})/\delta \in (1/2, 1)$ , region (iii) disappears.

Now, we show the effect of an increase in  $p^A$  on the hazard rate of policy implementation at the beginning of each period before the election. Interestingly the effect varies with the policy cost. As we can infer easily, if the policy cost is in the intermediate range such that equilibrium type (iii) applies, the effect of an increase in  $p^A$  on the hazard rate depends on the equilibrium choice. If we consider the equilibrium in which party  $A$  always implements the policy while party  $B$  always procrastinates, the hazard rate of policy implementation equals  $p^A$ , so that it increases as the degree of party  $A$ 's predominance increases. If we consider the equilibrium in which party  $A$

always procrastinates while party  $B$  always implements the policy, on the other hand, the hazard rate equals  $1 - p^A$ , so that an increase in the degree of party  $A$ 's predominance lowers the hazard rate. Since the effect of party  $A$ 's predominance is ambiguous simply due to the multiplicity of equilibrium when the policy cost is in the intermediate range, we shall focus on the investigation of the effects when the policy cost is small and when it is large. Interestingly, the effects are very different in the two cases.

First, we consider the case in which  $c < 2/(2 - \delta)$ . In this case,  $c$  is smaller than  $\underline{c}^B$  when  $p^A = 1/2$  so that both parties implement the policy when they are perfectly symmetric. Now, as  $p^A$  increases from  $1/2$ ,  $\underline{c}^B$  decreases and eventually becomes equal to  $c$  when  $p^A = [1 - (1 - \delta)c]/(\delta c)$ . The hazard rate of policy implementation drops from 1 to  $p^A$  at this point. The hazard rate increases as  $p^A$  increases further. This relationship is depicted in Figure 4. The effect of a change in the degree of party  $A$ 's predominance on the hazard rate of policy implementation is not monotonic. Rather, it shows the possibility that an increase in the degree of a party's predominance induces the predominated party to start procrastinating so that the hazard rate of policy implementation drops when the policy cost is small. Nonetheless, conditional on the predominant party being in office, the probability of implementation increases as party predominance increases.

Next, we turn to the case in which  $c > (2 - \delta)/[2(1 - \delta)]$ . In this case,  $c$  is greater than  $\bar{c}^A$  when  $p^A = 1/2$ . But as  $p^A$  increases,  $\bar{c}^A$  also increases and eventually becomes equal to  $c$  when  $p^A = (1 - \delta)(c - 1)/\delta$ . As Figure 5 shows, the hazard rate jumps up from 0 to  $p^A$  at this point and continues to increase as  $p^A$  increases. In this case, the hazard rate monotonically (though weakly) increases with  $p^A$ . If the policy cost is large, an increase in the degree of a party's predominance induces the predominant party to start implementing the policy so that it contributes to policy implementation.

We summarize the finding in the following proposition.

**Proposition 2** *Predominance of a party reduces the hazard rate of policy implementation in some cases if the policy involves a small cost. But the hazard rate (weakly) increases as the degree of a party's predominance increases if the policy involves a large cost.*

This proposition offers interesting predictions. A socially beneficial policy with a small policy

cost is likely to be implemented without much delay in a country where the degree of a party's predominance is small (or nil). One example that comes to mind is the United States, where relatively costless policy changes, such as food safety, were implemented quickly. On the other hand, a policy with a large policy cost is less likely to be implemented in a country with low degree of party predominance, and more likely to be implemented (sooner) in a country with a predominant party. A possible example of the latter case is China. Some pieces of anecdotal evidence appear to be consistent with these predictions. China's swift implementations of the rural health-insurance scheme and high-speed railway system mentioned in the Introduction are cases in point.

## 4.2 Implementation of a Divisible Policy: Gradual Implementation

Some policies that require an upfront cost but yield long-term benefits are divisible: trade liberalization, for example, can be implemented gradually over a stretch of time. The existence of a mixed-strategy equilibrium leads us to suspect that if the policy is divisible then each party may be willing to implement a fraction of the policy when in office given that the other does the same. Indeed, we show that if the policy is divisible there exists a stationary subgame perfect equilibrium with gradual implementation of the policy if the policy cost is in the intermediate range where the mixed-strategy equilibrium is supported (Type (iii) equilibrium introduced in Section 4.1). This "gradual implementation equilibrium" has a one-to-one correspondence with the mixed-strategy equilibrium.

Let us consider a stationary strategy profile such that whenever party  $i$  is in office, it implements a fraction  $a^i$  of the remainder of the policy of size  $\theta \in (0, 1]$ . Then, ruling party  $A$ 's expected welfare when  $A$  was in office in the last period and its expected welfare when  $B$  was in office in the last period can be written, respectively, as functions of  $\theta$ :

$$\begin{aligned} V^A(\theta) &= \phi^A[a^A\theta X_0^A + \delta V^A((1 - a^A)\theta)] + (1 - \phi^A)[a^B\theta \tilde{X}_0^A + \delta \tilde{V}^A((1 - a^B)\theta)], \\ \tilde{V}^A(\theta) &= (1 - \phi^B)[a^A\theta X_0^A + \delta V^A((1 - a^A)\theta)] + \phi^B[a^B\theta \tilde{X}_0^A + \delta \tilde{V}^A((1 - a^B)\theta)]. \end{aligned}$$

Let us guess that  $V^A(\theta)$  and  $\tilde{V}^A(\theta)$  are linear such that  $V^A(\theta) = \theta v^A$  and  $\tilde{V}^A(\theta) = \theta \tilde{v}^A$  where  $v^A$  and  $\tilde{v}^A$  are time-invariant. Then, these equations can be rewritten as

$$v^A = \phi^A[a^A X_0^A + (1 - a^A)\delta v^A] + (1 - \phi^A)[a^B \tilde{X}_0^A + (1 - a^B)\delta \tilde{v}^A], \quad (16)$$

$$\tilde{v}^A = (1 - \phi^B)[a^A X_0^A + (1 - a^A)\delta v^A] + \phi^B[a^B \tilde{X}_0^A + (1 - a^B)\delta \tilde{v}^A]. \quad (17)$$

It is immediate that (16) and (17) correspond term by term to (11) and (12) respectively, such that  $v^A = V^A$ ,  $\tilde{v}^A = \tilde{V}^A$ , and  $a^i = \sigma^i$  for  $i = A, B$ . As a consequence, we know that  $v^A$  can be written similarly to the expression for  $V^A$  in (13). The linearity of  $V^A(\theta)$  and  $\tilde{V}^A(\theta)$  with respect to  $\theta$  can easily be verified.

We know from the analysis of the mixed-strategy equilibrium that if  $a^A = \sigma^{A*}$  and  $a^B = \sigma^{B*}$ , where  $\sigma^{i*}$ , for  $i = A, B$ , is given in (14) and (15), then both parties are indifferent between implementing the policy and procrastinating, and hence it is ruling party  $i$ 's best response that it implements the fraction  $a^i$  of the remainder of the policy. We record this finding in the following proposition.

**Proposition 3** *Suppose that the policy is divisible. If the policy cost is in the intermediate range where the mixed-strategy equilibrium is supported, there also exists a stationary subgame perfect equilibrium in which the policy is gradually implemented. In this equilibrium, each party implements a constant fraction of the remainder of the policy when it is in office in such a way that the other party is indifferent between implementing and procrastinating when it is in office.*

## 5 Non-Stationary Equilibrium with Gradual Policy Implementation

We have shown that if the policy cost is large, neither party implements the policy in stationary subgame perfect equilibrium. This is the case where  $c > \bar{c}^A$ , or equivalently  $X_0^i < 0$ , for  $i = A, B$ , so that each ruling party would obtain negative welfare from implementing any positive fraction of the policy. As we infer from Figure 1, however,  $X_t^i$  may become positive if  $t$  is large enough so that each ruling party wishes even in such cases that the policy be implemented sometime in the future. To see this, we first rewrite  $X_t^i$  as

$$X_t^i = \beta_t^i \left[ \sum_{k=0}^{\infty} \frac{\beta_{t+k}^i}{\beta_t^i} - c \right]. \quad (18)$$

We can easily see from (6) that  $\beta_{k+1}^A/\beta_k^A$  (and also  $\beta_{k+1}^B/\beta_k^B$ ) converges to  $\delta$  as  $k$  goes to infinity.<sup>10</sup> Thus,  $\beta_{t+k}^i/\beta_t^i = \prod_{m=0}^{k-1} (\beta_{t+m+1}^i/\beta_{t+m}^i)$  approaches  $\delta^k$  as  $t$  increases, and hence the expression in square brackets on the right-hand side of (18) converges to  $\sum_{k=0}^{\infty} \delta^k - c$  as  $t$  goes to infinity. Since  $\sum_{k=0}^{\infty} \delta^k - c > 0$  under the assumption  $1/(1-\delta) > c$ , we have  $X_t^i = \beta_t^i \left[ \sum_{k=0}^{\infty} (\beta_{t+k}^i/\beta_t^i) - c \right] > 0$  as  $t$  exceeds a certain level. This contrasts sharply with the case of myopia. If the ruling parties are simply myopic (heavily discounting the future with geometric discounting), no ruling party wishes that the policy be implemented in the future if it would obtain negative utility from implementing it today. Time inconsistency arises precisely because the parties are present-biased when they are in office.

In this section, we examine if there exists a non-stationary subgame perfect equilibrium in which the policy is somehow implemented even when  $X_0^i < 0$ . We show that such a policy will not be implemented if the policy is indivisible, but it can be implemented gradually if the policy is divisible and the cost is relatively low. Gradual implementation of a divisible policy is supported by a trigger strategy such that a ruling party continues to implement a fraction of the policy as long as the parties have conformed whenever they were in office in the past.

If the policy is indivisible, the policy is never implemented if  $X_0^i < 0$  for  $i = A, B$ , or equivalently  $c > \bar{c}^A$ . It follows from  $X_0^i < 0$  that if the ruling party expects all future ruling parties to procrastinate, it should also procrastinate. That is, no ruling party wants to be the last to implement any positive fraction of the policy. If the policy is indivisible, therefore, no party will implement the policy because it would be the first and last ruling party to implement the policy if it implements the policy at all. The strategy profile in which neither party implements the policy, i.e.,  $a_t = 0$  for any  $t$ , is a subgame perfect equilibrium in such situations as we have seen. Thus, we have the following proposition.

**Proposition 4** *If the cost of the policy,  $c$ , is so high that  $X_0^i < 0$ , for  $i = A, B$ , and if the policy is indivisible, then the policy is never implemented even though it is socially beneficial.*

In this case, though the policy is socially beneficial, both parties procrastinate indefinitely.

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<sup>10</sup>Discounting between two consecutive periods tends to become similar to the geometric discounting far off in the future because the current incumbent advantage in elections diminishes as time goes by. Indeed, if there is no incumbent advantage in the first place, i.e.,  $s = 1$ ,  $\beta_1^i/\beta_0^i = \delta[p^i + (1-p^i)\alpha] < \delta$  but  $\beta_{k+1}^i/\beta_k^i = \delta$  for any  $k \geq 1$ .



If the policy is to be implemented at all when  $X_0^i < 0$ , it must be implemented gradually to ensure that every ruling party in the future enjoys nonnegative net benefits from its implementation. Moreover, the policy implementation process must continue indefinitely; otherwise, the ruling party that implements the policy to completion would suffer welfare loss from the part of the policy it implements. The following analysis presents such a gradual implementation equilibrium.

We shall show that when  $X_0^i < 0$  for  $i = A, B$ , a symmetric gradual implementation equilibrium exists if  $\sum_{k=0}^{\infty} X_k^i > 0$  for any  $i = A, B$ , i.e., the simple sum of all current and future utility flows is positive for both parties. The situation in which both  $X_0^i < 0$  and  $\sum_{k=0}^{\infty} X_k^i > 0$  simultaneously hold arises if  $c$  is relatively small in the high-cost range. The following lemma implies that  $X_k^i > 0$  for all  $k \geq 1$  when  $c$  takes a value such that  $X_0^i = 0$ . This in turn implies, by continuity of  $X_k^i$  with respect to  $c$ , that even if  $X_0^i < 0$ , it is possible that  $\sum_{k=0}^{\infty} X_k^i > 0$ .

**Lemma 4** *If  $\alpha < 1$ , then  $X_t^i > \beta_t^i X_0^i$  for any  $t \geq 1$ .*

The proof of Lemma 4 is relegated to the Appendix. The intuition is the following. Under the usual geometric discounting preferences such that  $\beta_t^i = \delta^t$ ,  $X_t^i$  would be equal to  $\beta_t^i X_0^i$ . Under the present-biased preferences, however, the current ruling party puts a disproportionately high weight on the cost incurred in the current period, and so  $X_0^i$  is disproportionately small.

Now, consider the stationary action profile (despite the non-stationary nature of the trigger strategy) that is symmetric between the two parties such that regardless of which party is in office, the ruling party in period  $t$  implements the fraction  $a_t = a(1-a)^t$  of the policy, for some constant  $a \in (0, 1)$ . According to this action profile, both parties implement the fraction  $a$  of the *remainder* of the policy whenever they are in office, and this process continues indefinitely. Consequently, the relevant welfare for the party in office in period  $t$  as evaluated in that period equals

$$\sum_{k=0}^{\infty} [a(1-a)^k X_k^i]. \quad (19)$$

**Lemma 5** *Suppose  $\sum_{k=0}^{\infty} X_k^i > 0$ . Then, there exists  $\bar{a} \in (0, 1)$  such that for any  $a \in (0, \bar{a})$ , the relevant welfare for the party in office in period  $t$  given by (19) is positive.*

**Proof:** We first notice that  $\sum_{k=0}^{\infty} (1-a)^k X_k^i$  converges to  $\sum_{k=0}^{\infty} X_k^i > 0$  as  $a \rightarrow 0$ . Thus, there

exists an  $\bar{a}$  such that for any  $a \in (0, \bar{a})$ ,  $\sum_{k=0}^{\infty} (1-a)^k X_k^i > 0$ , and hence  $\sum_{k=0}^{\infty} a(1-a)^k X_k^i > 0$ .

**Q.E.D.**

Can this gradual implementation scheme with  $a \in (0, \bar{a})$  be supported as a subgame perfect equilibrium? The answer is yes as the following strategy profile is subgame perfect.

$$a_t = \begin{cases} a(1-a)^t & \text{if there has been no deviation from } a_k = a(1-a)^k \text{ for all } k \leq t-1 \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

Hence, we obtain the following proposition.

**Proposition 5** *If the policy cost of the policy is sufficiently high that  $X_0^i < 0$  for  $i = A, B$ , but small enough that  $\sum_{k=0}^{\infty} X_k^i > 0$  for  $i = A, B$ , there exists a subgame perfect equilibrium in which every ruling party implements a constant fraction of the remainder of the policy so that the policy is implemented gradually.*

**Proof:** We show here that the strategy profile (20) is subgame perfect. Since indefinite procrastination is a subgame perfect equilibrium, we need only show that no ruling party has an incentive to deviate from the prescribed actions when there has been no deviation in the past. According to the prescribed action profile, if there has been no deviation, the ruling party in period  $t$  is to choose  $a_t = a(1-a)^t$ , receiving positive welfare from its action (Lemma 5). If it chooses some other level of  $a_t$ , on the other hand, the equilibrium path would switch to the “punitive equilibrium” of indefinite procrastination, making the present value of future utility flows zero. Since the ruling party’s utility from choosing a positive  $a_t$  in period  $t$  itself is negative, the discounted sum of utility flows would be non-positive if it chooses any  $a_t$  other than  $a(1-a)^t$ . Hence, the ruling party in period  $t$  is better off by conforming to the equilibrium action than choosing any other levels of  $a_t$ . Therefore, it will choose  $a_t = a(1-a)^t$  if there has been no deviation before period  $t$ . **Q.E.D.**

This “cooperative” equilibrium with the trigger strategy exists because both parties are present-biased. There are two reasons why the party in office may procrastinate. First, it may prefer the other party to implement the policy in the near future rather than implementing it itself. Second, the present value of utility flows obtained from the policy may become greater if the policy is carried out some time in the future due to hyperbolic discounting. The first reason was a primary cause of the mixed-strategy (and gradual implementation) equilibrium derived in the last subsection. The

second reason, on the other hand, is the primary cause of this gradual implementation equilibrium. Each party has an incentive to carry out part of the policy only when a large portion of it is sufficiently delayed so that the entire process of policy implementation yields a positive present value of utility flows.

## 6 Voters' Response to Policy Implementation

We have so far analyzed the problem of policy implementation assuming that voters are passive. The assumption of passive voters has simplified the analysis and enabled us to derive some important predictions of the model. In reality, however, ruling parties' decisions may well affect voting behavior. This is especially so when the electorate derives a large gain (or loss) from the policy when it is implemented. In this section, we extend the basic model to investigate the effect of this interaction between the electorate and political parties on the policy implementation. Specifically, we assume that if party  $i$  implements the policy, the probability of party  $i$  being re-elected increases from  $\phi^i$  to  $\hat{\phi}^i$  in all future periods after the implementation. That is, the electorate rewards the party that implements the socially beneficial policy by increasing its chance of being elected in all future periods.

Such responses by the electorate to the policy implementation naturally give each party more incentive to implement the policy. But at the same time, each party may have more incentive to procrastinate since the rival party is more likely to implement the policy. The following analysis clarifies the impact of the political interaction between the electorate and political parties on the policy implementation. Here, we only consider implementation of an indivisible policy.

As the analysis in Section 4 reveals, the key to deriving the subgame perfect equilibrium is to find a mixed-strategy equilibrium that arises when the policy cost  $c$  is in an intermediate range. To derive a mixed-strategy equilibrium, note that the probability that party  $i$  is re-elected increases from  $\phi^i$  to  $\hat{\phi}^i$  *after* it implements the policy. Therefore, the formulae (11) and (12) that show the expected welfare of party  $A$  at the beginning of each period *before* the policy implementation remain valid although the values of  $X_0^A$  and  $\tilde{X}_0^A$  change since they represent the net benefits that party  $A$  receives *after* the implementation of the policy. Consequently, the formulae for  $\sigma^{B*}$ , for

example, is not affected by the political interaction analyzed here.

Now, we rewrite the expression in (14) as

$$\sigma^{B*} = \frac{1 - \delta s + \delta^2(s - 1)}{\delta \left[ (1 - \phi^A) \frac{\tilde{X}_0^A}{X_0^A} - \phi^B \right] + \delta^2(s - 1)}$$

according to (21) in the Appendix. Then, we immediately realize that the political interaction affects  $\sigma^{B*}$  only through a change in  $\tilde{X}_0^A/X_0^A$ . Recall that  $X_0^i = 1 - c + \sum_{k=1}^{\infty} \beta_k^i$  and  $\tilde{X}_0^i = \alpha(1 - c) + \sum_{k=1}^{\infty} \tilde{\beta}_k^i$ , where  $\beta_k^i = \delta^k[p_k^i + (1 - p_k^i)\alpha]$  and  $\tilde{\beta}_k^i = \delta^k[1 - p_k^j + p_k^j\alpha]$  (when  $j \neq i$ ). It is intuitive and readily shown that  $p_k^i$  increases with  $\phi^i$  for  $i = A, B$ . Thus, the political interaction increases  $\beta_k^i$  and decreases  $\tilde{\beta}_k^i$ . This in turn means that the political interaction increases  $X_0^i$  and decreases  $\tilde{X}_0^i$ , so that  $\sigma^{B*}$  rises when voters reward parties for implementing the socially beneficial policy. It is obvious that  $\sigma^{A*}$  also rises under such circumstance.

The political interaction between the electorate and the political parties shifts up both schedules for  $\sigma^{A*}$  and  $\sigma^{B*}$  in Figure 2. Consequently, all threshold costs,  $\underline{c}^B$ ,  $\underline{c}^A$ ,  $\bar{c}^B$ , and  $\bar{c}^A$ , depicted in Figure 3, increase. Both parties tend to procrastinate less in the presence of the political interaction, so the region of  $c$  in which the policy is immediately implemented regardless of which party is in office expands while the region in which both parties procrastinate shrinks. The political interaction generally facilitates policy implementation by giving both parties more incentive to implement the policy. However, it is possible that the political interaction induces the dominant party  $A$  to procrastinate as it makes party  $B$  more willing to implement the policy. Such an outcome can arise if the regime shifts from (iv) to (iii) as a consequence of an increase in  $\bar{c}^B$  induced by the political interaction (see Figure 3). If this happens, the electorate's proactive voting behavior leads to an unintended consequence as it increases the expected delay in the policy implementation.

One possible interpretation of the outcome of the analysis of this section is that the procrastination problem can be mitigated if well-informed and far-sighted voters are more proactive in mobilizing more voters of their own type to vote as well as educating the public at large about the true costs and benefits of the policy itself. This would make each party's future probabilities of being elected more tightly tied to whether it implements the policy, thereby reducing the likelihood of procrastination.

## 7 Conclusion

We have presented a theory to explain government procrastination as a consequence of present-bias that in turn results from political uncertainty in a two-party political system. Present-bias arises because a party's probability of being elected in the future is less than 1, and because it puts more weight on the flow of net social benefit of the policy when it is in office than it would otherwise. As a result, the ruling party in a two-party political system often procrastinates about implementing socially beneficial policies that carry upfront costs but yield long-term benefits.

We find that there is an array of equilibria, which can be categorized according to the cost-benefit ratio of the policy. The procrastination problem tends to become more serious as the cost-benefit ratio gets higher. When the cost is relatively low, there is no procrastination problem. When the cost is in an intermediate range, there are various forms of procrastination, such as implementing the policy in each period with a probability less than one, or gradual implementation. When the cost is relatively high, the policy is procrastinated indefinitely in stationary equilibrium. However, due to the present-bias of the parties, there is a desire in both parties for the policy to be implemented in the sufficiently distant future. As a result, there exists a non-stationary subgame perfect equilibrium in which gradual implementation can be supported by a trigger strategy, if the cost is sufficiently small in the high-cost range.

Our theory predicts that a high degree of the predominance of a party may increase the likelihood of the government's procrastination about implementing a policy with a relatively small policy cost because the predominated party may procrastinate when it expects the predominant party to implement the policy. If the policy cost is large, on the other hand, the existence of the predominance of a party may facilitate the policy implementation. The higher the degree of predominance, the more future benefits the predominant party can reap from the policy implementation. As a result, the predominant party implements the policy that would not be implemented otherwise.

Finally, in the extension, we endogenize the probability of a party being elected. If the future probability of being elected increases for the party that implements the policy, the parties would have more incentive to implement the policy; this type of political interaction between the parties and the voters facilitates the policy implementation in general. Moreover, we have found that al-

though the threshold costs increase in favor of policy implementation, the equilibrium configuration is qualitatively the same as in the basic model. In other words, the results of the basic model are robust to the endogenization of the future probability of being elected.

## Appendix

### Proof of Lemma 1

First, we show that  $\sigma^{j^*}$ , for  $i = A, B$ , decreases with  $c$  when  $c$  is in an intermediate range such that  $\sigma^{j^*} > 0$ . We see from (14) and (15) that  $\sigma^{j^*} > 0$  if and only if  $X_0^i > 0$ . Thus, we rewrite  $\sigma^{j^*}$  as

$$\sigma^{j^*} = \frac{1 - \delta s + \delta^2(s - 1)}{\delta \left[ (1 - \phi^i) \frac{\tilde{X}_0^i}{X_0^i} - \phi^j \right] + \delta^2(s - 1)}, \quad (21)$$

where  $j \neq i$ . It is obvious from this formula that to show that  $\sigma^{j^*}$  decreases with  $c$ , we need only show that  $\tilde{X}_0^i/X_0^i$  increases with  $c$ .

To this end, we define a function that represents this ratio as  $g$  such that  $g(c) \equiv \tilde{X}_0^i(c)/X_0^i(c)$ , where  $X_0^i$  and  $\tilde{X}_0^i$  are represented as functions of  $c$ . The derivative of  $g$  equals

$$g'(c) = \frac{\sum_{k=1}^{\infty} (\tilde{\beta}_k^i - \alpha \beta_k^i)}{X_0^i(c)^2}. \quad (22)$$

To see the sign of  $g'(c)$ , we further define the function  $h$  by

$$\begin{aligned} h(\alpha) &\equiv \tilde{\beta}_k^i - \alpha \beta_k^i \\ &= \delta^k \left[ 1 - p_k^j + (p_k^j - p_k^i) \alpha - (1 - p_k^i) \alpha^2 \right]. \end{aligned}$$

Then, it is easy to see that  $h(0) = \delta^k (1 - p_k^j) > 0$ ,  $h(1) = 0$ , and  $h''^k (1 - p_k^i) < 0$ . That is, the function  $h$  is concave, taking a positive value at  $\alpha = 0$  and 0 at  $\alpha = 1$ . Thus, we know that  $h(\alpha) > 0$  for any  $\alpha \in [0, 1)$ , and hence  $g'(c) > 0$  for any  $c$  such that  $X_0^i(c) > 0$ . Thus, we have shown that  $\sigma^{i^*}$  decreases with  $c$  when it takes a value between 0 and 1. Moreover, we find from (22) that  $g$  is convex, i.e.,  $g''(c) > 0$ , since  $X_0^i(c)$  is decreasing in  $c$  while the numerator of the right-hand side of (22) is constant.

Next, we show that there exists a cost  $\underline{c}^j$  such that  $\sigma^{j^*} = 1$  when  $c = \underline{c}^j$  and a cost  $\bar{c}^j$  such that  $\sigma^{j^*} = 0$  such when  $c = \bar{c}^j$ .

It is readily shown from (21) that  $\sigma^{j^*} = 1$  if and only if

$$\frac{\tilde{X}_0^i}{X_0^i} = \frac{1 - \delta \phi^i}{\delta(1 - \phi^i)}.$$

Note that this value is greater than 1. To show that there is a cost  $c$  that satisfies this equation, we note that

$$\tilde{X}_0^i - X_0^i = (1 - \alpha)(c - 1) + \sum_{k=1}^{\infty} (\tilde{\beta}_k^i - \beta_k^i)$$

with

$$\tilde{\beta}_k^i - \beta_k^i = -(1 - \alpha)(s - 1)^k \leq 0, \quad (23)$$

where the last inequality holds since  $1 \leq s < 2$ . Thus, if  $c$  is small enough,  $\tilde{X}_0^i/X_0^i$  takes a value that is smaller than 1. Moreover,  $\tilde{X}_0^i/X_0^i$  goes to infinity as  $c$  goes up to a certain value that is smaller than  $1/(1 - \delta)$ , since  $g''(c) > 0$  and  $X_0^i$ , which is smaller than  $\sum_{k=0}^{\infty} \delta^k - c = [1/(1 - \delta)] - c$ , reaches 0 before  $c$  reaches  $1/(1 - \delta)$ . Therefore, we conclude that there exists a cost  $\underline{c}^j$  such that  $\sigma^{j*} = 1$  when  $c = \underline{c}^j$ .

On the other hand, we see from (14) and (15) that  $\sigma^{j*} = 0$  if and only if  $X_0^i = 0$ . Thus,  $\sigma^{j*} = 0$  if  $c = \bar{c}^i$  when we set  $\bar{c}^i = \sum_{k=0}^{\infty} \beta_k^i$ , since  $X_0^i = \sum_{k=0}^{\infty} \beta_k^i - c$ .

### Proof of Lemma 3

It suffices to show that  $\sigma^{A*} < \sigma^{B*}$ , given by (15) and (14), when  $\phi^A > \phi^B$ . It follows from (21) that we need only show indeed that

$$(1 - \phi^B) \frac{\tilde{X}_0^B}{X_0^B} - \phi^A > (1 - \phi^A) \frac{\tilde{X}_0^A}{X_0^A} - \phi^B.$$

Moreover, we can further appeal to the intrinsic symmetry of the two parties except for the difference in the probability of being re-elected to reduce the problem to showing that

$$f(\phi^A) \equiv (1 - \phi^A) \left( \tilde{X}_0^A/X_0^A \right) - (s - \phi^A), \quad (24)$$

where we have used  $\phi^B = s - \phi^A$  and  $(\tilde{X}_0^A/X_0^A)$  is viewed as a function of  $\phi^A$ , is decreasing in  $\phi^A$ .

To show that  $f(\phi^A) < 0$ , we first show that

$$\frac{\tilde{X}_0^A}{X_0^A} = \frac{\alpha(1 - c) + \sum_{k=1}^{\infty} \tilde{\beta}_k^A}{1 - c + \sum_{k=1}^{\infty} \beta_k^A}$$

decreases with  $\phi^A$ . It is easy to see that both  $\beta_k^A$  and  $\tilde{\beta}_k^A$  increases for any  $k$  as  $\phi^A$  increases. But it follows from (23) that  $\beta_k^A - \tilde{\beta}_k^A = (1 - \alpha)(s - 1)^k$  never changes with  $\phi^A$  so that  $\sum_{k=1}^{\infty} \tilde{\beta}_k^A$  and



$\sum_{k=1}^{\infty} \beta_k^A$  increase by the same amount as  $\phi^A$  increases. Since  $(\tilde{X}_0^A/X_0^A) > 1$  as shown in the proof of Lemma 1, this in turn implies that  $(\tilde{X}_0^A/X_0^A)$  decreases as  $\phi^A$  increases.

Now, we take a derivative of  $f$  defined by (24) to obtain

$$f'^A = \left(1 - \frac{\tilde{X}_0^A}{X_0^A}\right) + (1 - \phi^A) \frac{d(\tilde{X}_0^A/X_0^A)}{d\phi^A}.$$

We find that  $f'^A < 0$  since  $(\tilde{X}_0^A/X_0^A) > 1$ ,  $\phi^A < 1$ , and  $d(\tilde{X}_0^A/X_0^A)/d\phi^A < 0$ .

#### Proof of Lemma 4:

To prove Lemma 4, it suffices to show that  $\beta_{t+k}^i/\beta_t^i > \beta_k^i$ , or  $\beta_{t+k}^i > \beta_t^i \beta_k^i$ , for any  $t \geq 1$  and  $k \geq 1$ , since  $X_t^i = \beta_t^i \left[ \sum_{k=0}^{\infty} (\beta_{t+k}^i/\beta_t^i) - c \right]$  and  $X_0^i = \sum_{k=0}^{\infty} \beta_k^i - c$ . Indeed, we only show that  $\beta_{t+k}^A > \beta_t^A \beta_k^A$  since party  $B$ 's counterpart is obvious. Recall equation (5) and define

$$\begin{aligned} f(\alpha) &\equiv \alpha + (1 - \alpha) \{p^A + (1 - p^A)(s - 1)^{t+k}\} \\ &\quad - [\alpha + (1 - \alpha) \{p^A + (1 - p^A)(s - 1)^t\}] [\alpha + (1 - \alpha) \{p^A + (1 - p^A)(s - 1)^k\}]. \end{aligned}$$

It is easy to see that  $\beta_{t+k}^A > \beta_t^A \beta_k^A$  if and only if  $f(\alpha) > 0$ .

Now,

$$\begin{aligned} f(0) &= p^A + (1 - p^A)(s - 1)^{t+k} - [p^A + (1 - p^A)(s - 1)^t] [p^A + (1 - p^A)(s - 1)^k] \\ &= p^A(1 - p^A)[1 - (s - 1)^t][1 - (s - 1)^k] > 0, \end{aligned}$$

since  $0 \leq s - 1 < 1$ . In addition,  $f(1) = 0$ . Moreover, since

$$f''(\alpha) = -2[1 - p^A - (1 - p^A)(s - 1)^t][1 - p^A - (1 - p^A)(s - 1)^k] < 0,$$

the function  $f$  is concave. Thus, we have shown that  $f(\alpha) > 0$  for any  $\alpha \in [0, 1)$ .

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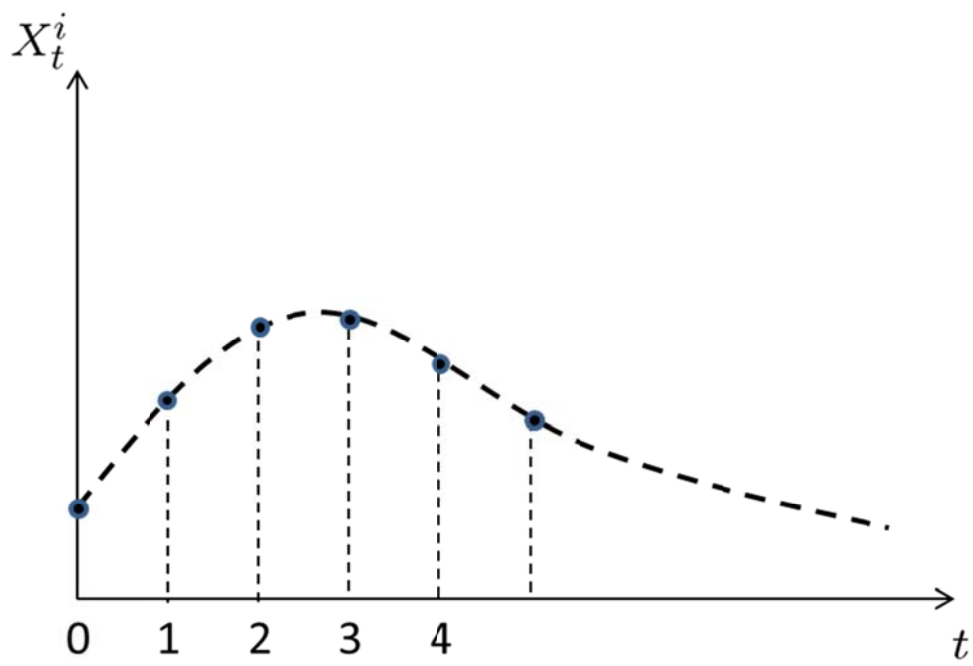


Figure 1. A Sequence of Present Discounted Welfare

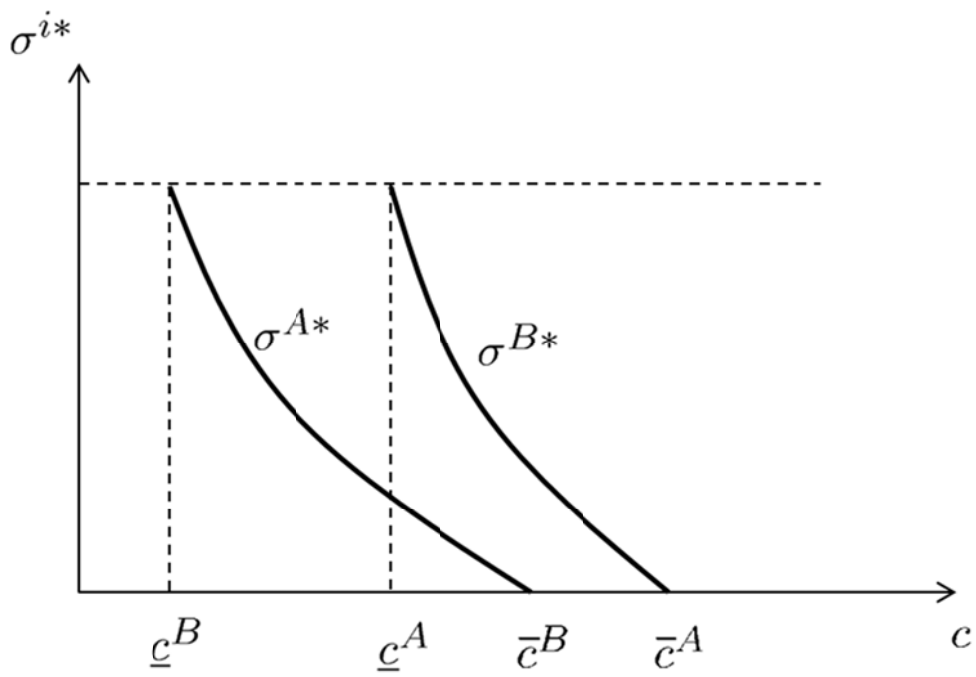


Figure 2. Implementation Cost and Mixed Strategy

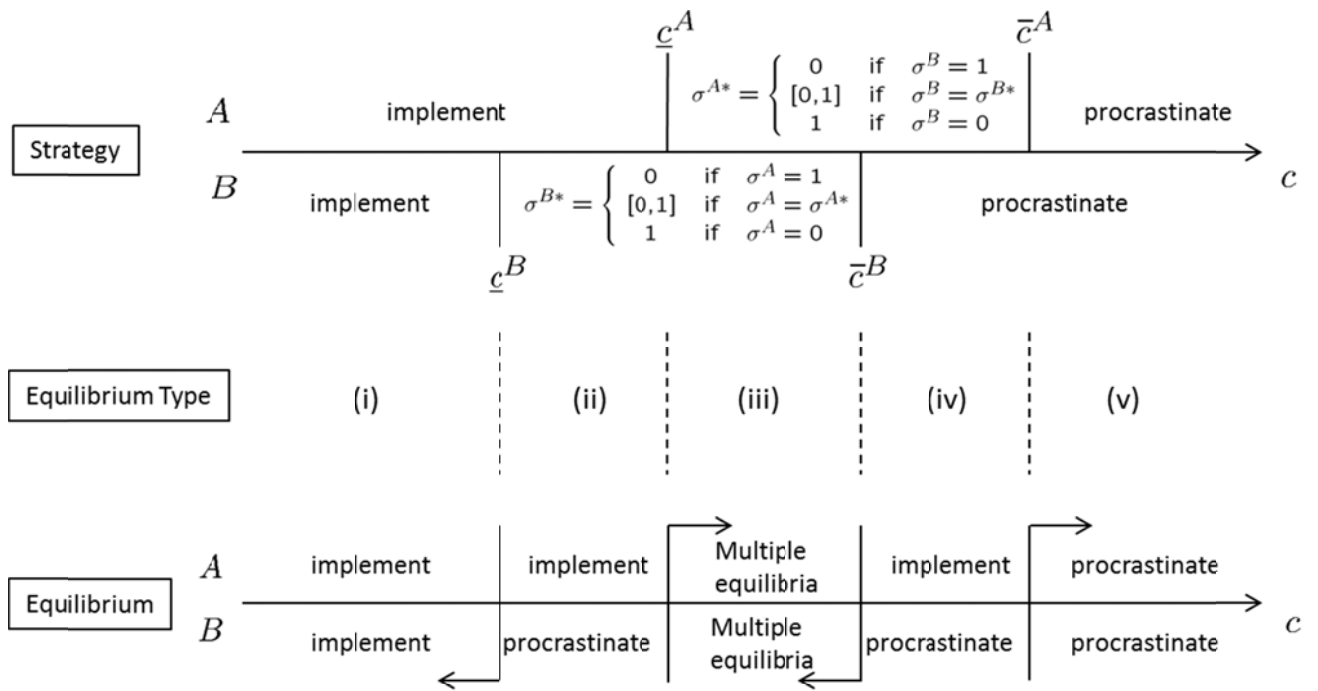


Figure 3. Best-Response Strategies and Subgame Perfect Equilibrium

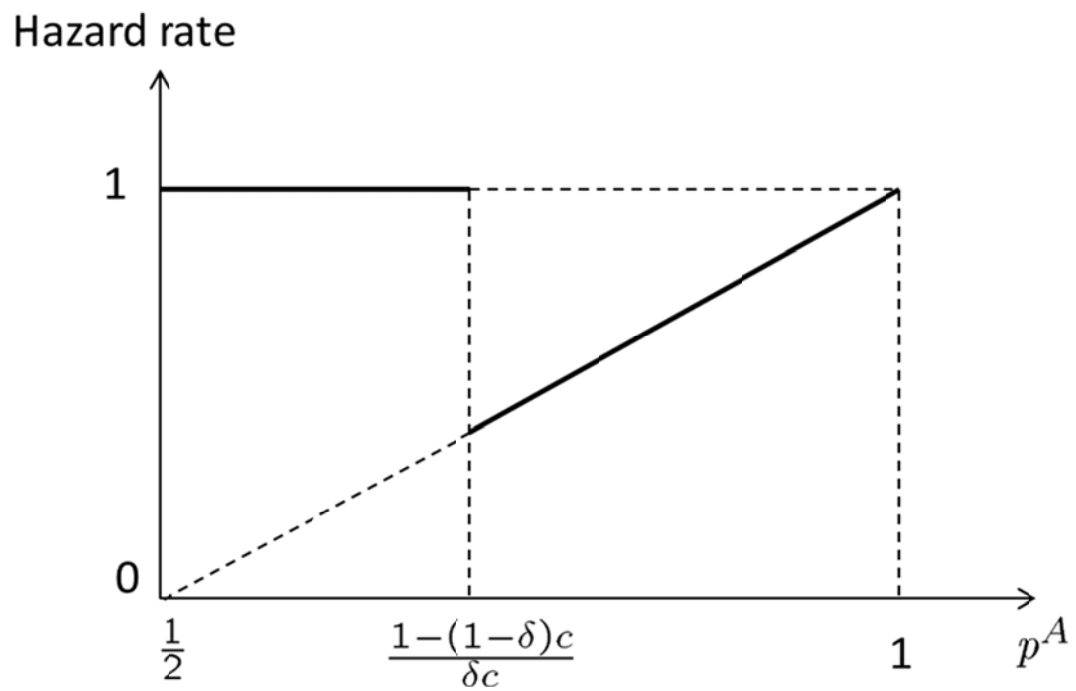


Figure 4. Party  $A$ 's Predominance and Hazard Rate:  $c$  is Small

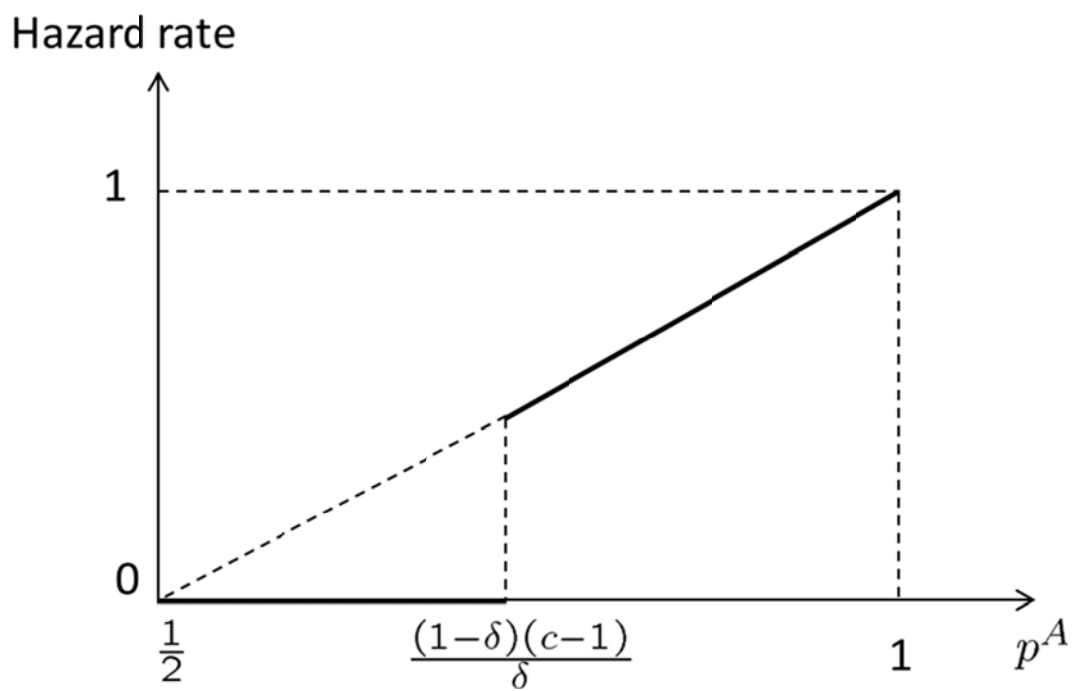


Figure 5. Party  $A$ 's Predominance and Hazard Rate:  $c$  is Large