Schumpeterian Growth with Gradual Product Obsolescence[1]

by

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Abstract

The model in this paper captures several important aspects of the real world: (i) gradual obsolescence of goods in the form of gradually declining net profit derived from each product until it is phased out; (ii) expanding variety of goods over time; and (iii) both dynamic and static internal increasing returns to scale of production. To eliminate the scale effect, Jones's specification that gives rise to a 'semi-endogenous' rate of innovation is adopted. The most interesting finding of the paper is that, when the research duplication effect is small (large) relative to the intertemporal knowledge spillover effect in R&D, the decentralized market delivers insufficient (excessive) obsolescence and allocates too little (much) labor to R&D, while a small subsidy (tax) to innovation is welfare-improving. All these results hold because the positive knowledge spillover externality overwhelms (is overwhelmed by) the negative research duplication externality.

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1 Introduction

Endogenous growth models that try to capture the Schumpeterian process of creative destruction usually feature the complete destruction of rents of firms producing lower quality products once a higher quality product is developed. (Aghion and Howitt, 1992; Grossman and Helpman 1991, chapter 4; Segerstrom, 1991). In these models, lower quality and higher quality products are perfect substitutes; therefore, lower quality products become obsolete immediately once a higher quality product is introduced into the market. On the other extreme, there are R&D-based endogenous growth models where growth is driven by the expansion of product variety (Romer, 1990; Grossman and Helpman, 1991, chapter 3.) In these models, although the market share of older goods are lowered by the introduction of a new variety, all goods will exist in the market forever.[2] Casual observation, however, suggests that goods become obsolete gradually as an economy grows.

We believe a richer Schumpeterian growth model should capture quality improvement, expansion of variety and gradual obsolescence of goods. With such a model, we would be able to endogenize not only the rate of innovation and growth, but also the rate of obsolescence and length of product cycles. In this paper, we modify an R&D-based growth model to capture these features and to analyze their effects on the properties of such a class of R&D-based model regarding policies, rate of innovation, the length of product cycles, welfare, etc.

The model in this paper captures the existence of both static and dynamic internal increasing returns to scale, as is well-documented in empirical studies. These increasing returns result from the interaction of a sunk cost of innovation, a quasi-fixed cost of production (overhead cost due to the presence of working capital) at each date, and a constant marginal cost of production. The utility function of consumers is CES, with newer goods carrying higher weights than older goods, because newer goods are more sophisticated. As a result, the instantaneous profit of a good declines over time, and is eventually phased out of the market when the firm's instantaneous gross profit is too small to cover the instantaneous quasi-fixed cost.

Endogenous growth models with gradual product obsolescence are not new. However, for the sake of tractability, the early models with this feature are mostly learning by doing

[2]The latter feature is basically an artifact of the Dixit-Stiglitz utility/production function, which, while being mathematically very tractable, has a 'love-of-variety' property.

models with perfect spillovers. The advantage of this type of model is it allows the authors to assume perfect competition in the product market (Stokey, 1988; Young, 1991.) Young's (1994) model is the first true hybrid of (Schumpeterian) R&D-based and learning by doing models, with imperfect competition in the product market and gradual phasing out of goods. His focus is on the interaction of invention and learning by doing as well as on how market size influences the bottleneck of growth. Our focus, however, is on the impacts of different types of taxation/subsidy and of exogenous changes within the framework of a purely R&D-based growth model.

Keppler (1996) depicts a rich model that explains the empirical regularities concerning how entry, exit, market structure and innovation vary from the birth of an *industry* through its maturity. In our model, we emphasize how optimizing behavior of agents and firms determines steady-state innovation, entry and exit of *products* in a broadly defined industry. We focus on how the length of the market-determined product cycle deviates from the social optimum and on the policy remedies.

This paper modifies an expanding-variety R&D-based growth model (Romer, 1990; Grossman and Helpman, 1991) to incorporate an asymmetrical CES specification for the index of consumption (with more recent products being more valuable) and an overhead cost of production at each date. These features allow for a limited lifespan of a product, which starts at the top-of-the-line when first introduced, and finishes its life as lowest quality. We have found a way to incorporate meaningful vintage products into a model of growth while keeping it tractable. We believe the steady state equilibrium captures the salient features of the product cycle — a product which starts as technologically innovative is gradually eclipsed by newer and more advanced products until it eventually disappears from the market altogether.

Jones (1995b) refutes the scale effect of the original frameworks of Romer (1990) and Grossman and Helpman (1991). Therefore, to eliminate the scale effect, we adopt Jones's (1995a) specification that leads to a 'semi-endogenous' rate of innovation.

Our model captures several important aspects of the real world: (i) there is gradual obsolescence of goods in the form of declining net profit derived from each product until it is phased out; (ii) there is expanding variety of goods over time; (iii) there are both dynamic and static internal increasing returns to scale of production. There are several Interesting results. First, an increase in any exogenous variable that positively affects the steady state rate of innovation leads to a(n) decrease (increase) in the equilibrium (socially optimal) fraction of obsolete products. Second, when the research duplication effect in R&D

is small (large) relative to the intertemporal knowledge spillover effect, the decentralized market delivers insufficient (excessive) obsolescence and allocates too little (much) labor to R&D, while a small subsidy (tax) to innovation is welfare-improving. These results hold because the positive knowledge spillover externality overwhelms (is overwhelmed by) the negative research duplication externality. When the research duplication effect in R&D is small (large) relative to the intertemporal knowledge spillover effect, the net external effect of obsolescence is positive (negative), and the product cycle is too short (long).

Section 2 lays down the main body of the model and solve for the endogenous variables. Section 3 compares the social optimum with the market equilibrium. Section 4 concludes the paper.

2 The Model

In this paper, we are only concerned with the 'balanced growth' steady state in which the rate of innovation is constant over time. There is one primary factor, labor, which can be used to undertake innovation (product development) or production of goods. Labor here, however, should be interpreted as skilled labor or 'human capital' that can work both in the production or R & D sector. There is one industrial sector (or, alternatively, M identical sectors) in which innovation takes the form of quality improvement. Each good contains a number of features each of which contributes to the utility function of a consumer. An innovation is defined as the development of a product that incorporates some additional feature to the most sophisticated existing product. In principle, there is an infinite number of potential features and potential goods that can be developed. There is no uncertainty in the R&D process.[3] At any given time, only a finite number of goods has been developed.

2.1 The Demand Side

Following Grossman and Helpman (hereinafter G-H) (1991, Ch. 3) we assume that a representative agent chooses total expenditure and consumption of goods at each date to maximize

[3]The absence of uncertainty in the R&D process, while unrealistic, simplifies the analysis and allows us to focus on the issues this paper wants to address.

the standard Ramsey intertemporal utility

$$W(t) = \int_t^\infty e^{-\rho(\tau-t)} log\, U(\tau) d\tau \tag{1}$$

subject to the intertemporal budget constraint[4]

$$\int_t^\infty e^{-r(\tau-t)} E(\tau) d\tau = \int_t^\infty e^{-r(\tau-t)} I(\tau) d\tau + A(t) \qquad for \qquad all \qquad t \tag{2}$$

where r = interest rate ; $U(\tau)$ = instantaneous utility at time τ ; $E(\tau)$ = instantaneous expenditure at τ ; $I(\tau)$ = instantaneous income at τ ; $A(t)$ = value of assets at t. At each date τ, the agent takes $A(\tau)$, $I(\tau)$, r and prices of goods as given. Instantaneous utility at time t is given by[5]

$$U(t) = \{\int_{n_o(t)}^{n(t)} [u(z)]^\alpha dz\}^{\frac{1}{\alpha}} \tag{3}$$

where goods in the continuum from index $n_o(t)$ to index $n(t)$ are the goods that exist in the market at time t (goods 0 to n_o have become obsolete at that date) and

$$u(z) \equiv u(x(z)) \equiv z^{\frac{1}{\beta}} x(z) \tag{4}$$

($0 < \alpha < 1$ and $0 < \beta$) is the subutility attributed to a good of index z. Good z contains one unit of each of the characteristics in the interval $(0, z)$, and $x(z)$ is the quantity of good z consumed. A good with a higher index has more features, and is considered 'more sophisticated'. The variable $n = n(t)$ denotes the most sophisticated good existing in the economy at time t.

Utility function (3) indicates that there is both a 'love of variety' and a 'love of sophistication' of goods. There is love-of-variety since the consumer is better off as long as more varieties are made available, even if the additional varieties are less sophisticated than the existing ones. There is love-of-sophistication because utility is increased by an increase in sophistication of any variety (measured by $z^{\frac{1}{\beta}}$ for good z), keeping the number of varieties (measured by $n - n_0$) unchanged. Although new goods are more sophisticated than old ones, the love-of-variety can be justified by the specialized function performed by each good. A good that has fewer features is assumed to be more specialized in (and thus better at) performing certain tasks than a more sophisticated good.

[4]The 'flow' version of this 'stock' equation is $I(t) - E(t) + rA(t) = \dot{A}(t)$.

[5]Alternatively, $U(t)$ can be regarded as quantity of final goods produced from a set of intermediate goods, with production function (3).

The dynamic optimization problem specified by (1), (2), (3) and (4) can be broken down into an intra-temporal optimization problem at time t of choosing $x(z)$ to maximize $U(t)$ subject to the instantaneous budget constraint (for given $n(t)$), and the intertemporal optimization problem of choosing a time path of $E(t)$ to maximize W subject to the demand function of $x(z)$ (determined by intra-temporal optimization on the demand side) and the prices of goods $p(z)$ (determined by intra-temporal optimization on the supply side).

The intra-temporal consumer optimization problem is

$$\underset{x(z)}{Max} \quad U(t)$$

s. t.

$$\int_{n_o(t)}^{n(t)} x(z)p(z)dz = E(t) \tag{5}$$

The intertemporal optimization problem will be solved after we have solved the instantaneous problems on the demand side and the supply side. We thereafter drop the time argument t for convenience, unless otherwise stated.

From the first order condition of the maximization problem (5), and some simple manipulation, we obtain the demand function $x(z)$ of good z,

$$x(z) = \frac{p(z)^{-\epsilon} z^{\gamma}}{\int_{n_o}^{n} p(s)^{1-\epsilon} s^{\gamma} ds} E \tag{6}$$

where $\epsilon = \frac{1}{1-\alpha} > 1$, and $\gamma = \frac{\alpha}{\beta(1-\alpha)}$. The parameter $\epsilon = \frac{1}{1-\alpha}$ is the elasticity of substitution between any two goods. So, ϵ increases with α. Other things being equal, the distribution of demand among goods of different degrees of sophistication is the result of consumers' compromise between the love-of-variety and the love of sophistication of goods. The greater α is, the less is the love of variety of goods. Similarly, the greater β is, the less is the love of sophistication.

2.2 The supply side

For tractability, all we need is to assume that there is a sufficient number of firms so that no single firm's action can significantly affect the denominator of (6). For ease of exposition, we adopt the stricter assumption that no two goods are produced by the same firm. Labor cost is the only variable cost of production. However, there are two non-variable costs: (i) before starting production, a firm needs to incur a sunk cost of product development; and

(ii) as long as the firm stays in business, the firm needs to incur a quasi-fixed cost at each date. This quasi-fixed factor can be thought of as such capital as testing equipment, quality control equipment, general purpose equipment (such as measuring devices, control devices), etc. For the sake of convenience, we shall simply call this quasi-fixed factor 'capital'. We assume that there is no depreciation of the capital and that there is a perfect second hand market for it. Therefore, a firm is indifferent between renting the capital or owning it for as long as it needs it. For ease of exposition, we can proceed with our analysis assuming that the firm purchases the capital when it starts producing a good and sells the capital when it stops producing the good. The quasi-fixed cost at each date is then the rental cost of capital, the value of which will be derived later.

Following Jones's (1995a) approach, we assume that the technology of research is constant returns to scale with respect to the quantity of labor devoted to R&D at the firm level. However, productivity of labor in R&D is positively affected by knowledge in society and negatively affected by the total quantity of labor devoted to R&D in society (due to negative externality of R&D duplication.) Hence, the number of new products developed by a firm is given by

$$\frac{1}{a}(l_n)n^{\phi}L_n^{\lambda-1},$$

where 'a' is a measure of unit labor requirement in R&D, $0 < \phi < 1$, $0 < \lambda < 1$, l_n is labor devoted to product development at the firm level, and L_n is the aggregate quantity of labor devoted to R&D in society. The term $L_n^{\lambda-1}$ captures the negative externalities occurring because of duplication in the R&D process. In equilibrium, when aggregating over all firms in the economy, $L_n = \sum l_n$. The term n^{ϕ} captures the positive externalities of knowledge on the productivity of labor employed in innovation, since n is a proxy for knowledge, as in Romer (1990).

Therefore, in equilibrium, the total number of new products developed in the economy at date t is given by

$$\dot{n} = \frac{1}{a}n^{\phi}L_n^{\lambda}. \tag{7}$$

The rate of innovation is given by $\frac{\dot{n}}{n} = \frac{1}{a}n^{\phi-1}L_n^{\lambda}$. In steady state, $\frac{\dot{n}}{n}$ is constant, therefore, $\lambda N = (1-\phi)g$, where $N \equiv \frac{\dot{L}_n}{L_n}$, which is exogenous in steady state, and g is the steady state rate of innovation. This impies that

$$g = \frac{\lambda N}{1-\phi}. \tag{8}$$

The steady state rate of innovation is higher when the rate of growth of research population N is higher, the negative externality of R&D duplication with respect to research population $1-\lambda$ is weaker, and the positive knowledge spillovers effect of R&D ϕ is stronger. Although profit-maximizing firms innovate out of profit motive, the rate of innovation, and therefore growth, is determined by variables that we usually regard as exogenous.

The labor productivity in innovation is $\frac{\dot{n}}{L_n} = \frac{1}{a} n^{\phi} L_n^{\lambda-1}$. If we define labor productivity in innovation as $\frac{m}{a}$, then

$$m = n^{\phi} L_n^{\lambda-1}$$

The growth rate of m is equal to the growth rate of labor productivity in innovation. From the previous equation, the steady state growth rate of m is $\frac{\dot{m}}{m} = \phi g - (1-\lambda)N = \phi g - (1-\phi)g(\frac{1-\lambda}{\lambda}) = \psi g$, where $\psi \equiv 1 - (\frac{1-\phi}{\lambda}) < 1$. We see that $\psi > 0$ only when $\lambda > 1-\phi$, that is, labor productivity in innovation grows in steady state only when negative externalities of research duplication are sufficiently weak and/or the positive externalities of knowledge spillovers are sufficiently strong.

Similarly, we assume that $\dot{K} = \frac{1}{b}(L_K) n^{\phi} L_n^{\lambda-1}$ in the aggregate, where 'b' is a measure of the unit labor requirement in production of capital; K is a measure of the quantity of capital in the economy, and L_K is the labor devoted to production of capital. Therefore, labor productivity in production of capital is $\frac{\dot{K}}{L_K} = \frac{1}{b} n^{\phi} L_n^{\lambda-1} = \frac{m}{b}$. That is, labor productivity in capital production is subject to the same negative and positive externalities as labor productivity in research. Note that assuming the same technology in production of blueprints and capitals is equivalent to saying that each innovation amounts to designing and building a 'factory' which enables an innovator firm to produce a differentiated product using labor as the variable factor and the capital in the 'factory' as a fixed factor. Moreover, a fraction $\frac{b}{a+b}$ of the 'factory' is re-usable and re-sellable in the second hand market. This will be made clear below.

Following the above modification to Romer (1990) and Grossman and Helpman (1991, ch.3), we obtain the total non-variable costs to be paid by an innovator-cum-producer before it starts production:

$$C_d = (a+b)\frac{w}{m} \tag{9}$$

where 'a' and 'b' are constants. As implied from the above discussion, $\frac{a}{m}$ is the unit labor requirement for product development, and $\frac{b}{m}$ is the unit labor requirement for production of capital.[6] w is the wage rate. Contrary to Romer (1990), the non-variable costs include not

[6] In equilibrium, the market value of a piece of capital is equal to the cost of its production because of the competitive market.

just the cost of developing a 'blueprint', but also the rental cost of a unit of capital at each date.

In addition to the non-variable costs, we assume that at each production date there is a constant labor requirement per unit of output (equal to one for all goods). Therefore, there are both static and dynamic internal increasing returns to scale of production.

The market structure is monopolistically Competitive, thus each firm has certain market power over the submarket of its good. Firms maximize the present discounted value of profits. Because of time separability of the intertemporal profit function, each firm chooses price, given the prices of other goods, to maximize instantaneous gross profit $\pi(z)$, subject to the demand function (6). Therefore, a producer solves

$$\underset{p(z)}{Max} \quad x(z)\{p(z) - c(z)\} \tag{10}$$

s.t. the demand function (6), where $c(z)$ is the cost per unit of output, which is equal to w because the unit labor requirement is equal to one. We obtain from the first order condition the mark-up pricing rule

$$p(z) = \frac{c(z)}{\alpha} = \frac{w}{\alpha} \tag{11}$$

where w = wage rate.

Using the results of the intra-temporal optimization problem, we obtain the first order condition for the intertemporal optimization as the Euler equation:

$$r = \rho + \frac{\dot{E}}{E}.$$

The above equation states that growth rate of E will be higher when consumers are less impatient (more willing to invest in the future), for any given r. We normalize by setting $w = m$. In other words, the wage paid to workers is always proportional to labor productivity in innovation (i.e. the number of new designs produced per worker). This implies that the price of a new design at any time is the same, meaning that a new design is the numeraire of the economy at any time. This normalization will imply that E/n is constant over time in steady state. We can then re-write the above equation as

$$r = \rho + g \tag{12}$$

2.3 Obsolescence

An innovator invests in R&D to develop a blue-print, which enables him to produce a new, differentiated product. It then earns the opportunity to make a stream of future profits. In steady state, the instantaneous profit (gross of capital cost) of a good diminishes over time because of the gradual introduction of more sophisticated goods. Free entry ensures that no firms can earn any discounted net profits.

On the balanced growth path, the steady state is characterized by $\frac{\dot{n}}{n} = \frac{\dot{n}_o}{n_o}$ so that n_o and n are in constant ratio with each other at all times. The number of variety $n - n_o$ therefore increases at the rate of $\frac{\dot{n}}{n}$. We define $\xi \equiv \frac{n_o}{n}$, the steady state fraction of goods that has become obsolete.

Using the demand function, production cost functions and mark-up pricing rule, namely (6) and (11), we can shown that the instantaneous gross profit of a firm producing good z, when the most sophisticated good is n (remember that $z < n$), is

$$\pi(z,n) = \pi(n,n)(\frac{z}{n})^{\gamma} \tag{13}$$

where

$$\pi(n,n) = \frac{E(\gamma+1)(1-\alpha)}{n(1-\xi^{\gamma+1})} \tag{14}$$

and instantaneous gross profit is defined as $\pi(z,n) \equiv x(z)[p(z) - c(z)]$.

In steady state, the initial instantaneous gross profit of any good when it is first introduced, $\pi(n,n)$, is constant over time since $\frac{E}{n}$ and $\frac{n_o}{n}$ are constant over time. According to (13), for given z, $\pi(z,n)$ decreases exponentially over time at a rate of γg in steady state, since n is increasing at a rate of g. Let γ be called the 'coefficient of obsolescence.' Therefore, the present discounted value (PDV) of the future gross profits of the firm falls at an exponential rate of $\gamma g + g + \rho$. The capital of a firm is resellable (to a new firm) in a perfectly competitive second hand market. Assuming no depreciation of the capital, its current value at time t is equal to $\frac{w(t)}{m(t)}b$, which is constant over time. Therefore, the PDV of the resale value of the capital of a firm falls at an exponential rate of $\rho + g$, which is slower than the rate of decline of the PDV of gross profits. It is therefore clear that, *at some point*, the firm would find it optimal to sell its capital rather than continue operating the firm. Specifically, a firm will stop producing a product as soon as the the marginal benefits of waiting (the time derivative of the resale value of the capital of the firm) are smaller than the marginal costs of waiting (the time derivative of the PDV of the future gross profits of the firm).

Let t_d be the time when good z is developed and t_o be the time when good z becomes obsolete; then $T = t_o - t_d$ is the age of good z at the time of obsolescence (i.e. T is the length of the 'product cycle'.) It can be shown that at steady state, when $\frac{\dot{n}}{n} = \frac{\dot{n}_o}{n_o} = g$,

$$\xi \equiv \frac{n_o}{n} = e^{-g(t_o - t_d)} = e^{-gT} \tag{15}$$

In Appendix B, it is shown that the marginal benefits of waiting is equal to the marginal costs of waiting when

$$\pi(n,n)e^{-(\gamma g+g+\rho)T} = (\rho+g)\frac{wb}{m}e^{-(\rho+g)T}$$

which implies

$$\pi(n,n)\xi^{\gamma} = (\rho+g)\frac{wb}{m}. \tag{16}$$

We shall refer to this as the obsolescence condition. The left hand side (LHS) is the instantaneous gross profit at the date of exit, while the right hand side (RHS) is the rental cost of capital at the date of exit. The firm exits when the instantaneous gross profit is just enough to cover the quasi-fixed cost at that date. This confirms the equivalence between the firm renting capital and its owning the capital for as long as the good is produced.

2.4 Zero Profit Condition of Innovators

Free entry without barriers implies that no firm can make any net profit. In equilibrium, therefore, the PDV of gross profits plus the PDV of the resale value of the capital of the innovator is equal to sum of innovation cost and purchase price of capital. Hence,

$$\int_{t_d}^{t_o} e^{-r(\tau - t_d)}\pi(n(t_d), n(\tau))d\tau + \frac{w}{m}be^{-r(t_o - t_d)} = (a+b)\frac{w}{m}$$

As shown previously, in steady state, $\pi(n(t_o), n(\tau))$ diminishes at an exponential rate of γg with respect to τ. This, together with (15), implies that the above equation is reduced to

$$\frac{\pi(n,n)}{(\rho+g+\gamma g)}(1-\xi^{\frac{\rho}{g}+1+\gamma}) + b\frac{w}{m}\xi^{\frac{\rho}{g}+1} = (a+b)\frac{w}{m}$$

This is a zero profit condition for the innovator. The first term on the LHS is the PDV of the stream of instantaneous gross profits of a firm, with the factor $1-\xi^{\gamma+1+\frac{\rho}{g}}$ accounting for the fact that the stream of profits will terminate upon obsolescence. The term $\rho+g+\gamma g$ is the discount factor that includes both the interest rate $\rho+g$ and the rate of obsolescence γg (which is in fact the rate of capital loss due to obsolescence.) The second term on the LHS is the PDV of the resale value of the capital of the firm.

Using (16) and the above zero profit condition, we obtain

$$\frac{\pi(n,n)}{\gamma g+g+\rho}\left(\frac{\rho+g+\gamma g\xi^{\gamma+\frac{\rho}{g}+1}}{\rho+g}\right)=(a+b)\frac{w}{m} \tag{17}$$

The term in parentheses on the LHS is greater than one, indicating that the PDV of total returns to the firm is greater than the PDV of gross profits from sales of goods, because of the positive resale value of the capital.

2.5 Market-determined value of ξ

Dividing (17) by (16) yields

$$\frac{\rho+g+\gamma g\xi^{\gamma+1+\frac{\rho}{g}}}{\rho+g+\gamma g}=\left(\frac{a+b}{b}\right)\xi^{\gamma} \tag{18}$$

where g is expressed as a function of λ, N and ϕ in (8). In Appendix A, it is shown that the above equation can be represented by a downward sloping curve DE in the (N,ξ) or (g,ξ) space, given a, b, λ and ϕ, as shown in Figure 1. That is, a greater N corresponds to a smaller value of ξ — a higher growth rate of the research population, which leads to a higher steady state rate of innovation, corresponds to a smaller equilibrium fraction of obsolete products in steady state. This relationship is shown in Figure 1 as the curve DE (for 'Decentralized Equilibrium'). Let us call the value of ξ under decentralized equilibrium ξ_{DE}. Hereinafter, all variables with subscript DE (SO) are associated with 'decentralized equilibrium' ('social optimum').[7]

The intuition is as follows. As g increases, the rate of obsolescence γg also increases, which implies that the instantaneous gross profit falls more rapidly. In order for the firm to break even, the initial gross instantaneous profit of a new product must increase. As a result, the instantaneous gross profit has to fall to a lower fraction of initial level when the firm exits from the market.[8] This fraction is exactly equal to ξ^{γ}, as shown in (16). Therefore, a higher g corresponds to a lower ξ in a decentralized market.

From (15), it is not clear whether the length of the product cycle T_{DE} increases or decreases as g increases (which leads to a decrease in ξ_{DE}). However, it is demonstrated by

[7] As $g\to\infty$, the value of ξ_{DE}, call it ξ_{min}, is defined by $\frac{1+\gamma\xi^{\gamma+1}}{1+\gamma}=(\frac{a+b}{b})\xi^{\gamma}$. It is hard to get a closed form solution to ξ_{min}, but it can be shown that $\xi_{min}\in([(\frac{1}{1+\gamma})(\frac{b}{a+b})]^{\frac{1}{\gamma}},[\frac{b}{(\gamma+1)a+b}]^{\frac{1}{\gamma}})$. The lower bound of ξ_{min} is shown in Figure 1.

[8] It is easier to understand this intuition if one pretends that r is independent of g. Although this is not true, the intuition is the same.

Mathematica that T_{DE} decreases as g increases for all the combinations of parametric values that have been tried. This result is consistent with casual observation that, in many sectors of the capitalistic economy, as the rate of innovation increases the length of product cycle shortens, while the fraction of obsolete products decreases.

A subsidy to innovation lowers the value of 'a' faced by firms. For given λ, N and ϕ, this leads to an increase in ξ_{DE} (and decrease in T_{DE}, according to (15)). Similarly, a tax on working capital increases the value of 'b' faced by firms and leads to an increase in ξ_{DE} and decrease in T_{DE}. See Figure 2 for the relationship between T_{DE} and g. Hence, we have

Result 1 *An increase in λ, N or ϕ leads to an increase in g, but a decrease in ξ_{DE} and T_{DE}. A subsidy to innovation, or tax on working capital, leads to an increase in ξ_{DE} and a decrease in T_{DE}.*

3 Social Optimum vs. Decentralized Equilibrium

We shall compare the socially optimal value of ξ with the decentralized market equilibrium value, then assess whether the competitive equilibrium carries too much inertia or momentum. We shall discuss the various externalities that lead to the divergence of the market equilibrium from the social optimum. The following analysis is similar to that of Grossman and Helpman (1991, Ch.3, pp.67-74).

To obtain the socially optimal path, we first of all solve a static problem of allocating resources to producing the various existing goods. Then we solve the dynamic problem of determining the growth of n over time. Let X be the aggregate output of manufactured goods. The static problem for the central planner is

$$\max_{x(z)} \qquad U = \{\int_{n_o}^{n} [z^{\frac{1}{\beta}} x(z)]^{\alpha} dz\}^{\frac{1}{\alpha}} \qquad s.t. \qquad \int_{n_o}^{n} x(z) dz \leq X$$

The Lagrangean of the above optimization problem is

$$\mathcal{L} = \{\int_{n_o}^{n} [z^{\frac{1}{\beta}} x(z)]^{\alpha} dz\}^{\frac{1}{\alpha}} + \lambda[X - \int_{n_o}^{n} x(z) dz] \tag{19}$$

where λ is the shadow value of one unit of labor.

From the first order condition, it is straightforward to show that

$$\frac{x(z)}{X} = \frac{z^{\gamma}}{\int_{n_o}^{n} z^{\gamma} dz} \tag{20}$$

Substituting this into the expression for U above, we obtain

$$U = X(\frac{n^{\gamma+1} - {n_o}^{\gamma+1}}{\gamma+1})^{\frac{1-\alpha}{\alpha}} \tag{21}$$

From the above equation, $log\, U$ expressed as a function of X, ξ and n is

$$log\, U(X, \xi, n) = (\frac{1-\alpha}{\alpha})[(\gamma+1)log\, n - log\,(1+\gamma) + log(1-\xi^{\gamma+1})] + logX \tag{22}$$

Equation (9) says that the unit labor requirement for product development is $\frac{a}{m}$ and the unit labor requirement for the production of new capital is $\frac{b}{m}$. Since each firm needs one unit of capital for production, the quantity of new capital required is $\dot{n} - \dot{n}_0$, because old capital supplied from obsolete firms satisfies part of the demand in the market. Therefore, total labor in the economy devoted to R&D and the production of new capital at each date is equal to $(\frac{a}{m})\dot{n} + (\frac{b}{m})(\dot{n} - \dot{n}_0) = L_n + \frac{b}{a}L_n(1-\xi)$. It follows that

$$X = L - L_n - \frac{b}{a}L_n(1-\xi) \tag{23}$$

In the dynamic problem, the central planner maximizes $\int_0^\infty e^{-\rho\tau} log\, U(X, \xi, n) d\tau$ subject to the labor constraint (23) and R&D technology (7). The current value Hamiltonian of this dynamic optimization problem is

$$\mathcal{H} = (\frac{1-\alpha}{\alpha})[(1+\gamma)log\, n - log\,(1+\gamma) + log(1-\xi^{\gamma+1})] + log[L - L_n - \frac{b}{a}L_n(1-\xi)] + \theta(\frac{1}{a}n^{\phi}L_n^{\lambda}) \tag{24}$$

where θ is the costate variable that represents the (current) shadow value of variety, L_n and ξ are the control variables, and n is the state variable.

3.1 Optimal ξ

In Appendix C, it is shown that the optimal ξ is characterized by [9]

$$\frac{\lambda g}{\rho + (1-\phi)g} = [\frac{a + b(1-\xi)}{b(1-\xi^{\gamma+1})}]\xi^{\gamma}, \tag{25}$$

where g as a function of λ, N, and ϕ is given in (8). The above equation can be represented by an upward sloping curve in the (N, ξ) or (g, ξ) space, given a, b, λ and ϕ. That is, an increase in the growth rate of research population, which leads to an increase in the

[9] Refer to G-H (1991, Ch.3, p.71) for a similar derivation procedure.

steady state rate of innovation, corresponds to an increase in the optimal fraction of obsolete products in society. This curve is shown in Figure 1 as SO (for 'Social Optimum').[10]

Intuitively, an increase in ξ phases out older goods today but makes it possible to allocate more resources to develop sophisticated goods for the future. Therefore, it trades off less variety today for more variety tomorrow. As g gets larger, the positive marginal effect of ξ on tomorrow's variety increases (since knowledge stock grows faster, making labor more efficient in R&D in future) relative to its negative marginal effect on today's variety. Hence, the optimal ξ increases with g.

From (15), we know that an increase in g, which leads to an increase in ξ_{SO}, implies that T_{SO} decreases. That is, the optimal length of the product cycle falls as g increases, as shown in Figure 2. From (25) and (15), we conclude that ξ_{SO} increases and T_{SO} decreases as 'a' decreases or 'b' increases. Hence we have

Result 2 *An increase in λ, N or ϕ leads to an increase in g, an increase in ξ_{SO} and a decrease in T_{SO}. A decrease in 'a' or an increase in 'b' leads to an increase in ξ_{SO} and a decrease in T_{SO}.*

In Figure 1, we see that, for given a, b, λ and ϕ, since DE is downward sloping and SO is upward sloping in the (N, ξ) or (g, ξ) space, they intersect exactly once as long as the asymptotic level of SO is greater than that of DE. This is true when $\frac{\lambda}{1-\phi} > \frac{(a+b)}{a(\gamma+1)}$.[11] That is, if $\frac{\lambda}{1-\phi}$ is sufficiently large, then $\xi_{DE} > \xi_{SO}$ when g is small, and $\xi_{DE} < \xi_{SO}$ when g is large. This means the market-determined product cycle is too short (or there is 'excessive momentum' in the obsolescence process) when g is small and the market-determined product cycle is too long (or there is 'excessive inertia' in the obsolescence process) when g is large. Figure 2 basically describes the same phenonmenon. Note that although both the DE and SO curves are downward sloping in Figure 2, the SO curve is above the DE curve for $N < N^*$ but below DE for $N > N^*$, for the reason depicted in Figure 1: for the same g, $\xi_{DE} > \xi_{SO}$ iff $T_{SO} > T_{DE}$, since $\xi = e^{-gT}$.

[10] As $N \to \infty$, the value of ξ_{SO}, call it ξ_{max}, is defined by $\frac{\lambda}{1-\phi} = [\frac{a+b(1-\xi)}{b(1-\xi^{\gamma+1}}]\xi^{\gamma}$. It is hard to obtain a closed form solution to ξ_{max}, but we can show that $\xi_{max} \in ([\frac{b\lambda}{b\lambda+(a+b)(1-\phi)}]^{\frac{1}{\gamma}}, [\frac{b\lambda}{b\lambda+a(1-\phi)}]^{\frac{1}{\gamma+1}})$. The upper bound of ξ_{max} is shown in Figure 1.

[11] This sufficient condition is derived from subtracting the upper bound of ξ_{min} for DE from the lower bound of ξ_{max} for SO obtained from footnotes 7 and 10 and restricting the expression to be greater than zero.

The finding that there is excess momentum (excess inertia) in the obsolescence process when the steady state rate of innovation is small (large) makes sense: In the extreme case that the rate of innovation is zero, it is certainly optimal to have no obsolescence of goods, because of the love-of-variety of consumers. However, there will be obsolescence in the market because the instantaneous gross profits of the least sophisticated goods are too small to cover the quasi-fixed cost of production.

The intuition for the divergence between ξ_{DE} and ξ_{SO} is as follows. There are three externalities as a firm exits from the market: (a) A negative externality on today's variety (*current variety effect*) — the number of today's older variety decreases; (b) a positive externality on profits of other firms today (*profit-creation effect*) — the profits of firms that remain in the market increase because there is less competition; and (c) a positive externality on future variety (*future variety effect*) — the firm's exit makes it possible to allocate more resources to develop more sophisticated variety in the future. Such allocation results in positive externality when $\frac{\lambda}{1-\phi}$ is sufficiently large.[12] When $g = 0$, externality (c) is zero, but (a) dominates (b), and so the market delivers too much obsolescence.[13] As g grows, (c) becomes more significant, since the labor efficiency in R&D increases in the future as the knowledge stock grows faster. Eventually, (c) overwhelms (a), the market begins to deliver too little obsolescence.

Hence, we have

Result 3 *Suppose* $\frac{\lambda}{1-\phi}$ *is sufficiently large that* $\frac{\lambda}{1-\phi} > \frac{(a+b)}{a(\gamma+1)}$. *For given a and b, when g is sufficiently small,* $\xi_{DE} > \xi_{SO}$ *and* $T_{DE} < T_{SO}$ *(i.e. the product cycle is too short in the decentralized equilibrium). When g is sufficiently large,* $\xi_{DE} < \xi_{SO}$ *and* $T_{DE} > T_{SO}$ *(i.e. the product cycle is too long in the decentralized equilibrium).*

On the other hand, if $\frac{\lambda}{1-\phi}$ is too small, the DE and SO curves will not cross, and ξ_{SO} is always less than ξ_{DE}. This is true when $\frac{\lambda}{1-\phi} < \frac{a}{(1+\gamma)(a+b)-b}$.[14] In other words, when the degree

[12] A larger λ implies that the externality resulted from duplication of research is less serious. A larger ϕ implies that the externality resulted from intertemporal knowledge spillovers is stronger. Both effects tend to increase the net positive externality of resource allocation to R&D. To understand more about the externality of allocation of labor to R&D, see the discussion above Result 5.

[13] Note that when all goods carry equal weights in the Dixit-Stiglitz CES utility function, (a) and (b) just cancel each other. (See Grossman and Helpman 1991, p.70.) In that case, an exit always results in a positive externality.

[14] This sufficient condition is obtained from imposing the restriction that the upper bound of ξ_{max} for SO

of research duplication is too strong, the market always delivers too much obsolescence, because it has too much incentive to allocate resources from production of older goods to development of new sophisticated products. Hence, we have

Result 3A If $\frac{\lambda}{1-\phi} < \frac{a}{(1+\gamma)(a+b)-b}$, then $\xi_{SO} < \xi_{DE}$ for all g.

Therefore, if $\frac{\lambda}{1-\phi} < 1$, it is possible that $\xi_{SO} < \xi_{DE}$ when γ gets sufficiently small.

3.2 Fraction of labor allocated to Research $\frac{L_n}{L}$

With regard to the social optimum, we have, from (31) in Appendix C,

$$\frac{a}{b}(\frac{L}{L_n} - 1) = (\frac{\alpha}{1-\alpha})[\frac{1-\xi^{\gamma+1}}{(\gamma+1)\xi^{\gamma}}] + (1-\xi). \quad (26)$$

where ξ varies directly with g as given in (25).

It is clear from the equation that an increase in optimal ξ is associated with an increase in optimal $\frac{L_n}{L}$. From Result 2, we know that an increase in λ, ϕ or N is accompanied by an increase in the optimal ξ. Therefore, we conclude that an increase in λ, ϕ or N leads to an increase in the socially optimal $\frac{L_n}{L}$.

With regard to the decentralized equilibrium, Appendix D shows:

$$\frac{a}{b}(\frac{L}{L_n} - 1) = (\frac{\alpha}{1-\alpha})[\frac{1-\xi^{\gamma+1}}{(\gamma+1)\xi^{\gamma}}](\frac{\rho+g}{g}) + (1-\xi). \quad (27)$$

Using Mathematica, we find that the RHS of the above equation decreases with g, noting that ξ is a function of g as given in (18). Together with (8), we can conclude that an increase in λ, ϕ or N leads to an increase in $(\frac{L_n}{L})_{DE}$. A subsidy on innovation decreases the value of 'a' faced by firms, but not the actual resource requirements for R&D. Therefore, the value of 'a' on the RHS of (18) decreases, leading to a higher ξ_{DE}, for given g, which in turn leads to a higher $(\frac{L_n}{L})_{DE}$, according to equation (27). This is shown in Figure 3.

Therefore, we have

Result 4 *(Numerical) An increase in λ, N or ϕ leads to an increase in g, and increases in $(\frac{L_n}{L})_{SO}$ and $(\frac{L_n}{L})_{DE}$. A subsidy on innovation leads to an increase in $(\frac{L_n}{L})_{DE}$.*

be smaller than the lower bound of ξ_{min} for DE obtained from footnotes 7 and 10.

Jones (1995a) suggests that $(\frac{L_n}{L})_{SO}$ diverges from $(\frac{L_n}{L})_{DE}$ for three reasons: (a) When $\phi > 0$, $(\frac{L_n}{L})_{SO} - (\frac{L_n}{L})_{DE}$ increases with ϕ, reflecting a positive externality due to the knowledge spillovers from previous research. (b) When $1-\lambda > 0$, $(\frac{L_n}{L})_{SO} - (\frac{L_n}{L})_{DE}$ decreases with $1-\lambda$, which reflects a negative externality due to duplication of research by different firms. (c) $(\frac{L_n}{L})_{SO} - (\frac{L_n}{L})_{DE}$ increases with α, reflecting monopoly price markup, which causes too little labor to be devoted to research. Jones (1995a) finds that $(\frac{L_n}{L})_{SO} < (\frac{L_n}{L})_{DE}$ when $1-\lambda$ is sufficiently close to one (i.e. λ close to zero) and $(\frac{L_n}{L})_{SO} > (\frac{L_n}{L})_{DE}$ when $1-\lambda$ is sufficiently close to zero (i.e. λ close to one).

Our findings are similar. From (26) and (27), we can deduce that a sufficient condition for $(\frac{L_n}{L})_{SO} > (\frac{L_n}{L})_{DE}$ when g is large is $\frac{\lambda}{1-\phi} > \frac{(a+b)}{a(\gamma+1)}$.[15] Recalling that $g = \frac{\lambda N}{1-\phi}$, we conclude that $(\frac{L_n}{L})_{SO} > (\frac{L_n}{L})_{DE}$ when $\frac{\lambda}{1-\phi}$ is sufficiently large. This is exactly the condition that guarantees that there is insufficient obsolescence in the decentralized equilibrium when g is large.[16] Using Mathematica, we also confirm that $(\frac{L_n}{L})_{SO} < (\frac{L_n}{L})_{DE}$ when $\frac{\lambda}{1-\phi}$ is sufficiently small. Therefore, we have

Result 5 *The decentralized market allocates too little labor to research when* $\frac{\lambda}{1-\phi} > \frac{(a+b)}{a(\gamma+1)}$ *and g is sufficiently large. It is also demonstrated numerically that the decentralized market allocates too much labor to research when* $\frac{\lambda}{1-\phi}$ *is sufficiently small.*

Results 4 and 5 are summarized by Figure 3.

Let us now consider how government policies can improve welfare. Note that there are now two market-determined variables the values of which can diverge from the social optima — the fraction of labor allocated to R&D and the fraction of obsolete goods. Therefore, any government remedy has to take care of the divergence in both variables. When $\frac{\lambda}{1-\phi} > \frac{(a+b)}{a(\gamma+1)}$ and g is sufficiently large that $\xi_{DE} < \xi_{SO}$, a small subsidy to innovation (which lowers the value of 'a' faced by firms) will increase the market-determined ξ and push it towards the optimal value, according to Result 1. It also increases the market-determined $\frac{L_n}{L}$ towards the socially optimal value provided that $\frac{\lambda}{1-\phi}$ is sufficiently large, according to Result 4 and 5. Since there are no other distortions that cause the decentralized outcome to deviate from the social optimum, a small innovation subsidy will be welfare-improving when $\xi_{DE} < \xi_{SO}$.

[15] $(\frac{L_n}{L})_{SO} \to 0$ and $(\frac{L_n}{L})_{DE} \to 0$ as $N \to 0$. On the other hand, a sufficient condition for $(\frac{L_n}{L})_{SO} - (\frac{L_n}{L})_{DE} > 0$ as $N \to \infty$ is for $\xi_{SO} > \xi_{DE}$ as $N \to \infty$, a sufficient condition of which is $\frac{\lambda}{1-\phi} > \frac{(a+b)}{a(\gamma+1)}$, which is derived in footnote 11.

[16] This confirms our point given earlier (just before Result 3) that the future variety effect of obsolescence is positive only when the allocation of labor to research results in positive externality.

When $\frac{\lambda}{1-\phi}$ is large but N is sufficiently small that $\xi_{DE} > \xi_{SO}$, a small tax on innovation (which increases 'a' faced by firms) will decrease the market-determined ξ, pushing it towards the socially optimal value, but it will also decrease the market determined $\frac{L_n}{L}$, pushing it away from the optimal value when $\frac{\lambda}{1-\phi}$ is sufficiently large. Therefore, the welfare consequence of such a effect is not obvious. However, when $\frac{\lambda}{1-\phi}$ or N is sufficiently small, g is very small, and $\xi_{DE} - \xi_{SO}$ is very large, but the deviation between $(\frac{L_n}{L})_{SO}$ and $(\frac{L_n}{L})_{DE}$ approaches zero. Therefore, it is clear that a small tax on innovation is welfare-improving when $\frac{\lambda}{1-\phi}$ or N is sufficiently small. Hence we have

Result 6 *A small subsidy to innovation is welfare-improving when* $\frac{\lambda}{1-\phi} > \frac{(a+b)}{a(\gamma+1)}$ *and* g *is sufficiently large that* $\xi_{DE} \leq \xi_{SO}$*. A small tax on innovation is welfare-improving when* $\frac{\lambda}{1-\phi}$ *or* N *is sufficiently small.*

Hence, a subsidy to innovation is not always welfare-improving, as in Grossman and Helpman (1991). In fact, even when $(\frac{L_n}{L})_{SO} > (\frac{L_n}{L})_{DE}$, a subsidy to innovation is not always welfare-improving, because the subsidy could lead to further divergence between ξ_{DE} and ξ_{SO}, the welfare effect of which is negative. The latter welfare effect can dominate when g is small.

Note that a large $\frac{\lambda}{1-\phi}$ indicates that the research duplication effect is small relative to the intertemporal knowledge spillovers effect. The main message of this paper is that Results 3, 3A, 5 and 6 show the crucial importance of this parameter in determining the divergence between the optimal and equilibrium variables in the economy. Recalling that $g = \frac{\lambda N}{1-\phi}$, we have the following corollary from these Results:

Corollary 1 *When* $\frac{\lambda}{1-\phi}$ *is sufficiently large (small),* $\xi_{SO} > (<)\xi_{DE}$*,* $(\frac{L_n}{L})_{SO} > (<)(\frac{L_n}{L})_{DE}$ *and a small subsidy (tax) to innovation is welfare-improving.*

It is also worth mentioning the role of γ and N. A corollary from Results 3, 5 and 6 is

Corollary 2 *When* N *and* γ *are sufficiently large,* $\xi_{SO} > \xi_{DE}$*,* $(\frac{L_n}{L})_{SO} > (\frac{L_n}{L})_{DE}$ *and a small subsidy to innovation is welfare-improving. This is true regardless of the value of* $\frac{\lambda}{1-\phi}$*.*

Intuitively, when consumers value new goods more (i.e. γ increases), and knowledge accumulates more quickly (i.e. g increases as N increases), the positive external effects of allocating labor to R&D increase. When γ and N are sufficiently large, $(\frac{L_n}{L})_{SO} > (\frac{L_n}{L})_{DE}$. This type of externality has not received much attention in the literature.

4 Discussion and Caveats

Our model is a Schumpeterian growth model that captures a rich array of features in the real world: endogenous innovation, gradual obsolescence of goods, expanding variety of goods, and both dynamic and static internal increasing returns to scale in production. The most interesting finding is that, when the research duplication effect in R&D is small (large) relative to the intertemporal knowledge spillover effect, the decentralized market delivers insufficient (excessive) obsolescence and allocates too little (much) labor to R&D, and a small subsidy (tax) to innovation is welfare-improving. All these results hold because the positive externality due to knowledge spillover overwhelms (is overwhelmed by) the negative externality due to research duplication. We also find that when consumers' love-of-sophistication of new goods is stronger and the growth rate of research personnel is higher, the positive externality of R&D increases. This externality due to consumers' aspiration of new products has not received enough attention in the literature.

There are a couple of caveats. Certain features of the model are not totally consistent with the real world. In the model, the newest product has the highest market share and the oldest one the smallest. In the real world, it is usually the products of medium age that have the largest market share because firms need to acquire enough production experience in order to attain the full potential of an innovation. We believe we can modify our model to capture this by allowing for a learning or reputation-building period for any new product, or some lagged response of consumers to better products which has just been introduced into the market.

Second, in this model, the number of active products in the market increases exponentially. Some may not find this very plausible. This feature is an artifact of the specific functional form of knowledge spillover *a la* Romer (1990) and Jones (1995a). There are other utility functions that can generate sustained exponential growth in steady state, yet with a fixed number of goods in the market. We believe the same qualitative results will be obtained.[17]

[17]For example, we could use the instantaneous utility function

$$U = \{\int_{n_o}^{n} [\lambda^z x(z)]^\alpha dz\}^{\frac{1}{\alpha}}$$

In that case, there will be sustained exponential growth with a constant $n - n_o$ in steady state even when there is no knowledge spillover of the Romer type.

For future work, we can extend this model to a North-South product cycle trade model where the South imitates Northern products which may or may not have become obsolete in the North. Depending on the technological capability of the South relative to that of the North, different patterns of international division of labor between the North and South in production of goods can emerge. We can then evaluate whether countries still unambiguously gain from trade, what determine the world rate of innovation and the effects of certain tax policies on growth and welfare.

Appendix

A The slope of the DE Curve

It is straightforward to show that $\frac{\partial}{\partial\xi}(\frac{\rho+g+\gamma g\xi^{\gamma+1+\frac{\rho}{g}}}{\xi^{\gamma}}) = \frac{\gamma(\rho+g)(\xi^{\gamma+\frac{\rho}{g}+1}-1)}{\xi^{\gamma+1}} < 0$. Also, $\frac{\partial LHS_{18}}{\partial g} < 0$, as shown below.

Let $\phi(\xi, g) = \frac{\rho+g+\gamma g\xi^{\gamma+1+\frac{\rho}{g}}}{\rho+g+\gamma g}$. Hence,

$$\frac{\partial\phi(\xi, g)}{\partial g} = \frac{1}{(\gamma g + g + \rho)^2}\{(\gamma g+g+\rho)[\gamma\xi^{\frac{\rho}{g}+1+\gamma}+\gamma\xi^{\frac{\rho}{g}+1+\gamma}(\frac{\rho}{g})log(\frac{1}{\xi})]-(\rho+g+\gamma g\xi^{\frac{\rho}{\gamma}+1+\gamma})(\gamma+1)\}$$

which tends to $\frac{-1}{\gamma g+g+\rho}$ as ξ tends to 1.

Moreover,

$$\frac{\partial}{\partial\xi}\frac{\partial\phi(\xi, g)}{\partial g} = \frac{\partial}{\partial g}\frac{\partial\phi(\xi, g)}{\partial\xi} = \frac{\partial}{\partial g}\gamma\xi^{\frac{\rho}{g}+\gamma} = \gamma\xi^{\frac{\rho}{g}+\gamma}(\frac{\rho}{g^2})log(\frac{1}{\xi}) > 0 \qquad \text{for } 0 < \xi < 1.$$

Hence, $\frac{\partial\phi(\xi,g)}{\partial g} < 0$ for the entire relevant range of ξ, viz. $\xi \in [0, 1]$, as shown in Figure A1.

Therefore, by the implicit function theorem, $\frac{d\xi}{dg} < 0$ for $0 < \xi < 1$ and $g \neq \infty$.

B Optimal Timing of Obsolescence

Let B be the PDV of the benefit of reselling the capital of the firm at time t_0; and C be the PDV of the opportunity cost of reselling the capital at t_0. Because of perfect competition in the capital market, the resale value of the capital is always equal to the cost of producing it, which is wb/m, which in turn equals b, because of the normalization $w = m$. The value of C is the PDV of the worth of the firm. Therefore,

$$B = be^{-(\rho+g)(t_o-t_d)}$$

and

$$C = \int_{t_o}^{\infty} \pi(n, n)e^{-(\rho+g+\gamma g)(\tau-t_d)}d\tau$$
$$= \pi(n, n)[\frac{e^{-\gamma g(t_o-t_d)}e^{-(\rho+g)(t_o-t_d)}}{\gamma g + g + \rho}]$$

where $\pi(n,n)$ is invariant over time, as shown in (14).

In steady state,

$$\frac{\partial B}{\partial t_o} = -[(\rho+g)be^{-(\rho+g)(t_o-t_d)}]$$

$$\frac{\partial C}{\partial t_o} = -\pi(n,n)e^{-(\rho+g+\gamma g)(t_o-t_d)}.$$

$$\frac{\partial^2 B}{\partial t_o^2} = (\rho+g)^2 be^{-(\rho+g)(t_o-t_d)}$$

$$\frac{\partial^2 C}{\partial t_o^2} = (\gamma g+g+\rho)\pi(n,n)e^{-(\rho+g+\gamma g)(t_o-t_d)}.$$

It is clear that at the time when $\frac{\partial B}{\partial t_o} = \frac{\partial C}{\partial t_o}$, $B > C$ and $\frac{\partial^2(B-C)}{\partial t_o^2} < 0$. Therefore, both the first and second order conditions of maximization are satisfied. This implies that the optimal timing of selling the capital of the firm is given by the equation immediately above (16).

C Derivation of Social Optimum

Recall that

$$\mathcal{H} = (\frac{1-\alpha}{\alpha})[(1+\gamma)log\,n - log\,(1+\gamma) + log(1-\xi^{\gamma+1})] + log[L - L_n - \frac{b}{a}L_n(1-\xi)] + \theta(\frac{1}{a}n^{\phi}L_n^{\lambda})$$

From the above current value Hamiltonian, the first order condition with respect to n (i.e. $\dot{\theta} = \rho\theta - \frac{\partial\mathcal{H}}{\partial n}$) can be written as

$$\frac{\dot{\theta}}{\theta} = \rho - (\frac{1-\alpha}{\alpha})(\frac{\gamma+1}{n\theta}) - \frac{\phi}{a}n^{\phi-1}L_n^{\lambda} \quad (28)$$

Following Grossman and Helpman (1991, Chapter 3), we define $M \equiv \theta n$, the shadow value of the total amount of variety. Substitute for $\frac{\dot{\theta}}{\theta}$ from $\frac{\dot{\theta}}{\theta} = \frac{\dot{M}}{M} - \frac{\dot{n}}{n}$ and $g = \frac{1}{a}n^{\phi-1}L_n^{\lambda}$, we obtain

$$\dot{M} = [\rho + (1-\phi)g]M - (\frac{1-\alpha}{\alpha})(\gamma+1).$$

The only path of M that satisfies the above equation as well as the transversality condition is

$$M = (\frac{1-\alpha}{\alpha})[\frac{\gamma+1}{\rho+(1-\phi)g}]. \tag{29}$$

Substituting for $g = \frac{1}{a}n^{\phi-1}L_n^{\lambda}$ and for $M = n\theta$, the first order condition with respect to L_n (i.e. $\frac{\partial \mathcal{H}}{\partial L_n} = 0$) can be written as

$$\frac{\lambda M g}{L_n} = \frac{1+\frac{b}{a}(1-\xi)}{L-L_n-\frac{b}{a}L_n(1-\xi)} \tag{30}$$

where M is given by (29). The RHS is the marginal effect of L_n on $log\, X$. The LHS of the above equation is the marginal effect of L_n on the shadow value of new varieties $\theta\dot{n}$.

The first order condition with respect to ξ (i.e. $\frac{\partial \mathcal{H}}{\partial \xi} = 0$) is

$$\frac{\frac{b}{a}L_n}{L-L_n-\frac{b}{a}L_n(1-\xi)} = (\frac{1-\alpha}{\alpha})[\frac{(\gamma+1)\xi^{\gamma}}{1-\xi^{\gamma+1}}] \tag{31}$$

Eliminating $\frac{L_n}{L-L_n-\frac{b}{a}L_n(1-\xi)}$ between the previous two equations, we get (25).

D Equilibrium fraction of labor allocated to R&D

Labor is inelastically supplied at a quantity of L at each date. Therefore, from (11), total labor content of the sum of all the instantaneous variable costs of production at each date is:

$$X = \int_{n_o}^{n} \frac{c(z)}{w}x(z)dz = \frac{\alpha}{w}\int_{n_o}^{n} p(z)x(z)dz = \frac{\alpha}{w(1-\alpha)}\int_{n_o}^{n} \pi(z,n)dz.$$

Substituting (13) into the above equation, and simplifying, we obtain

$$X = \frac{\alpha}{(1+\gamma)(1-\alpha)}(1-\xi^{\gamma+1})\pi(n,n)\frac{n}{w}, \tag{32}$$

which implies that

$$\pi(n,n) = (\frac{1-\alpha}{\alpha})\frac{w}{n}\Phi, \tag{33}$$

where $\Phi = \frac{(1+\gamma)X}{1-\xi^{\gamma+1}}$ and $\frac{\Phi}{n}$ is the labor content of the instantaneous variable cost of production for good n at the time of innovation. In other words, instantaneous gross profit of good n is simply a mark-up factor $\frac{1-\alpha}{\alpha}$ times the wage bill at that date.

To obtain the reduced form of the obsolescence condition (16), we simply substitute (33) into (16) then invoke (23) and the normalization equation $w = m$:

$$L - L_n - \frac{b}{a} L_n (1 - \xi) = b \frac{n}{m} \left(\frac{1 - \xi^{\gamma+1}}{\xi^{\gamma}}\right)\left(\frac{\alpha}{1 - \alpha}\right)\left(\frac{1}{1 + \gamma}\right)(\rho + g)$$

Since $g = \frac{L_n^{\lambda}}{a n^{1-\phi}}$ and $m = \frac{n^{\phi}}{L_n^{1-\lambda}}$, it can be shown that $\frac{n}{m} = \frac{L_n}{ag}$. Substituting this into the above equation, and re-arranging, we obtain

$$\frac{a}{b}\frac{L}{L_n} - \frac{a}{b} = \left(\frac{\alpha}{1 - \alpha}\right)\left[\frac{1 - \xi^{\gamma+1}}{(\gamma + 1)\xi^{\gamma}}\right]\left(\frac{\rho + g}{g}\right) + (1 - \xi)$$

which is the same as (27).

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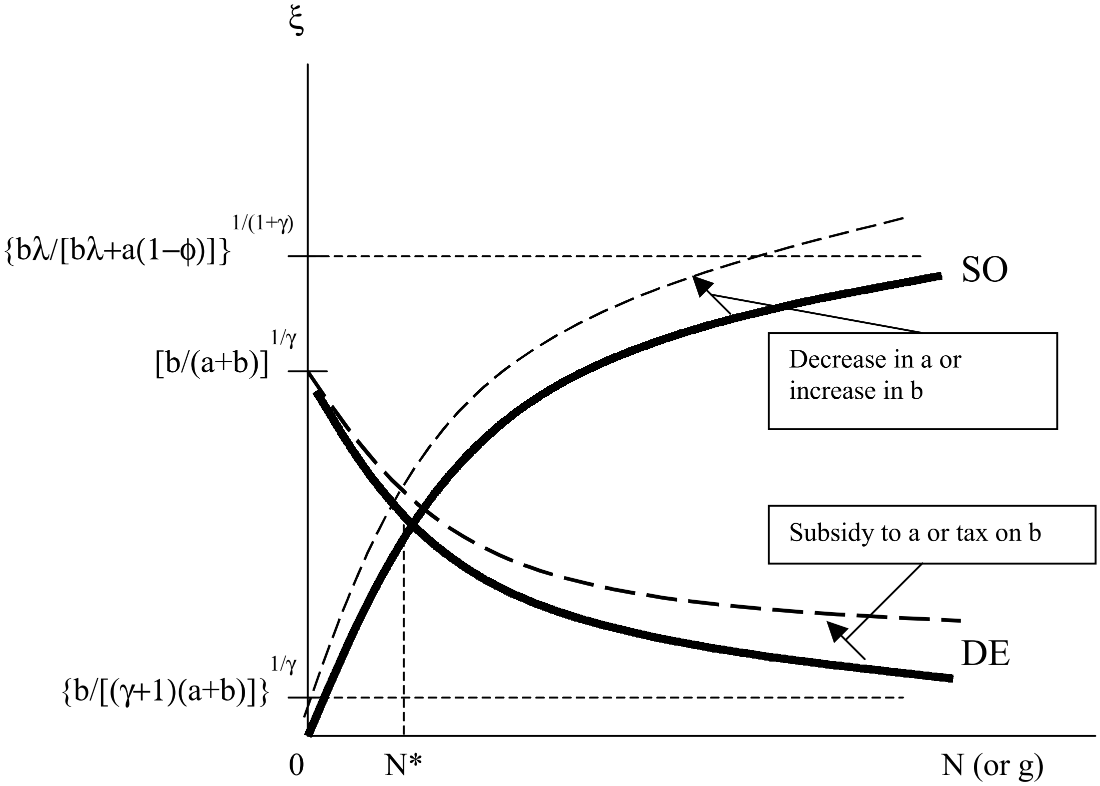

Figure 1: Relationship between the SO and DE Curves

Given a, b, λ, φ, γ, the curves will cross if $\lambda/(1-\phi) > (a+b)/[a(\gamma+1)]$.

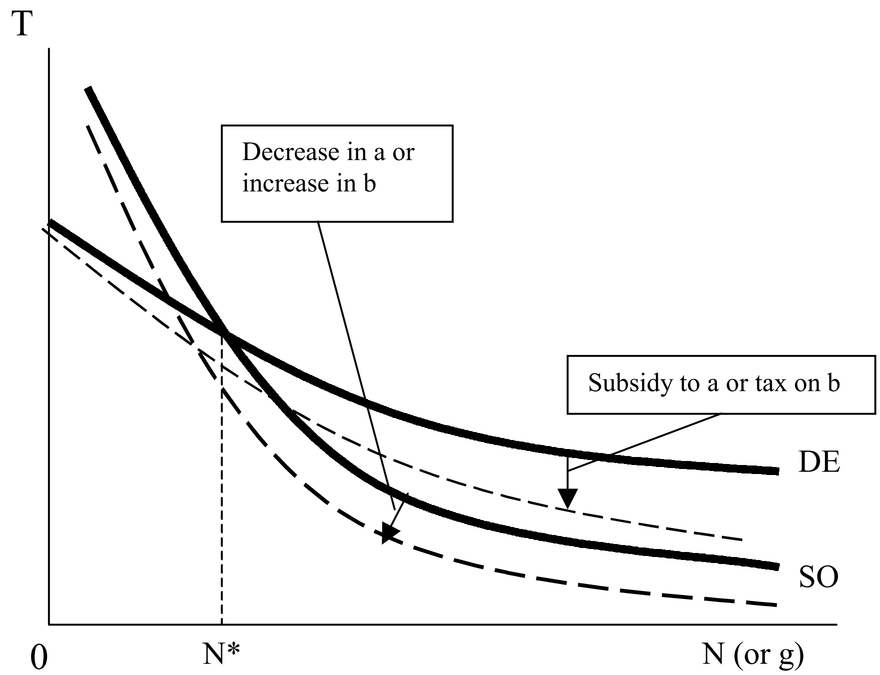

Figure 2: Comparison of lengths of product cycle between social optimum and decentralized equilibrium

Given a, b, λ, ϕ, γ, the curves will cross if $\lambda/(1-\phi) > (a+b)/[a(\gamma+1)]$. The DE curve is found to be downward sloping based on numerical simulation by Mathematica.

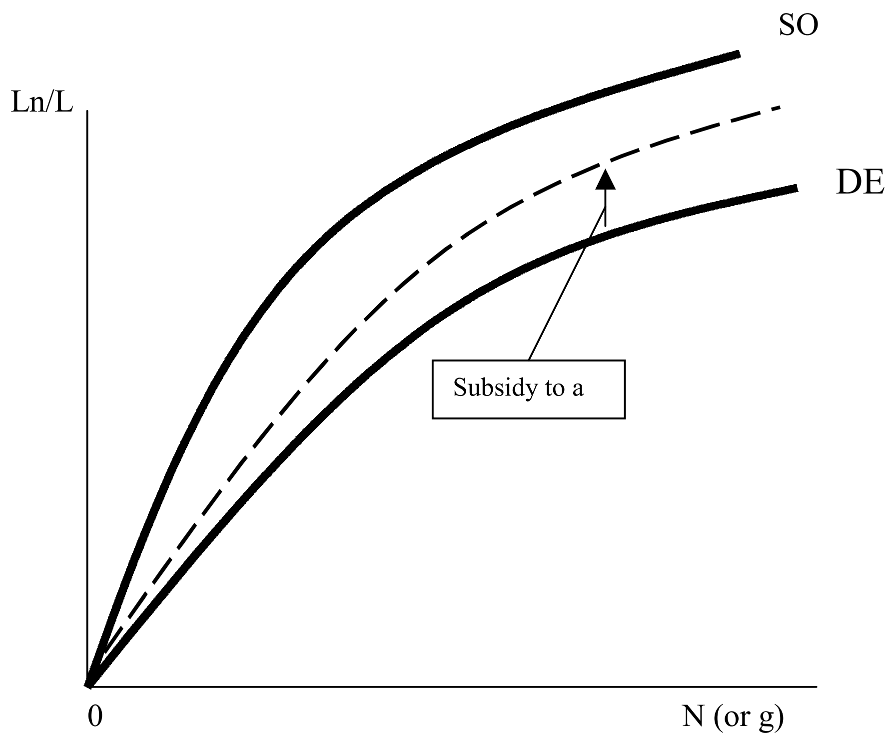

Figure 3: Comparison of labor allocation to R&D between social optimum and decentralized equilibrium

a, b, λ, ϕ, γ are given. The DE curve is found to be upward sloping based on numerical simulation using Mathematica. The SO curve is above the DE curve when $\lambda/(1-\phi)$ is sufficiently large.

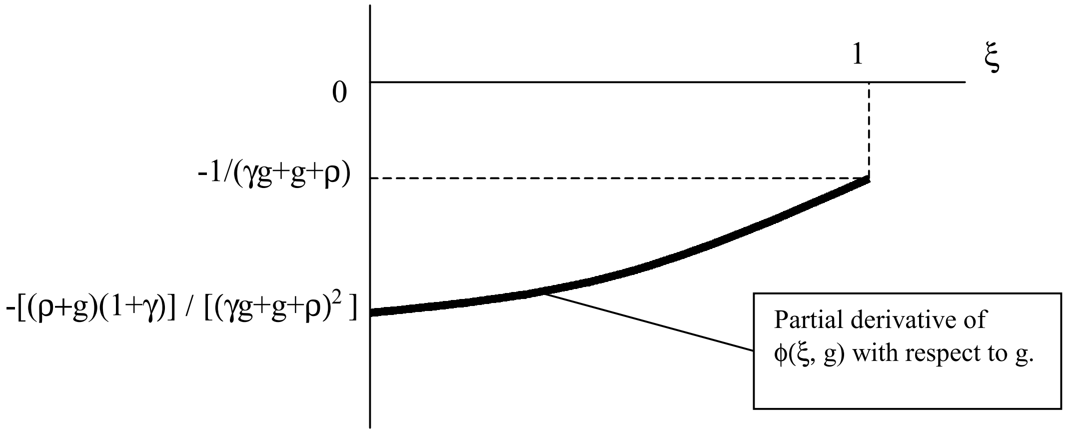

Figure A1: Diagram related to the proof in Appendix A